FLAT SLABS

Introduction

Common practice of design and construction is to support the slabs by beams and support the beams by columns. This may be called as beam-slab construction. The beams reduce the available net clear ceiling height. Hence in warehouses, offices and public halls some times beams are avoided and slabs are directly supported by columns. This type of construction is aesthetically appealing also. These slabs which are directly supported by columns are called Flat Slabs. Fig. 1.1 shows a typical flat slab.

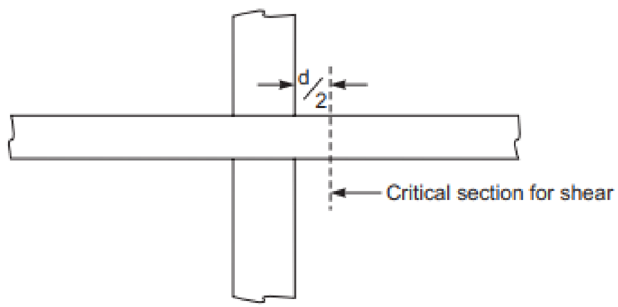


Fig. 1.1 *A typical flat slab (without drop and column head)*

The column head is some times widened so as to reduce the punching shear in the slab. The widened portions are called column heads. The column heads may be provided with any angle from the consideration of architecture but for the design, concrete in the portion at 45º on either side of vertical only is considered as effective for the design [Ref. Fig. 1.2]

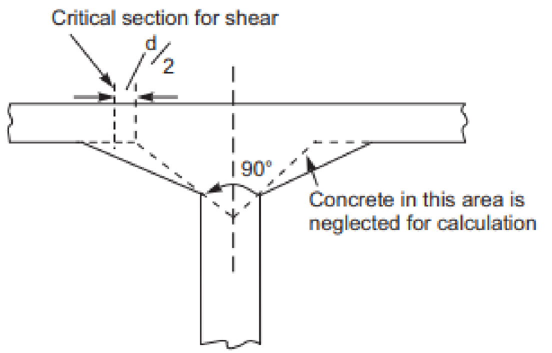


Fig. 1.2 *Slab without drop and column with column head*

Moments in the slabs are more near the column. Hence the slab is thickened near the columns by providing the drops as shown in Fig. 1.3. Sometimes the drops are called as capital of the column. Thus we have the following types of flat slabs:

i. Slabs without drop and column head (Fig. 1.1).
ii. Slabs without drop and column with column head (Fig. 1.2).
iii. Slabs with drop and column without column head (Fig. 1.3).
iv. Slabs with drop and column head as shown in Fig. 1.4.

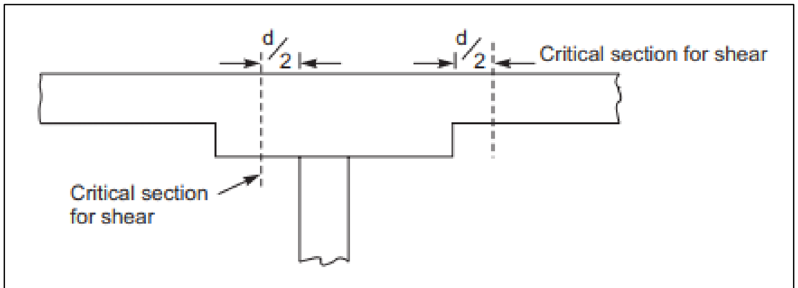


Fig. 1.3 *Slab with drop and column without column head*

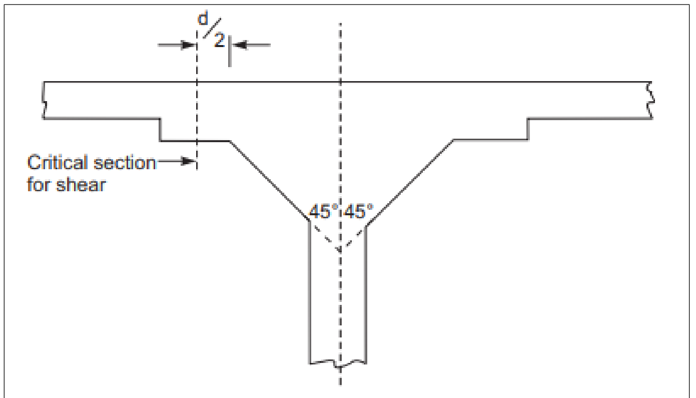


Fig. 1.4 *Slab with drop and column with column head*

The portion of flat slab that is bound on each of its four sides by centre lines of adjacent columns is called a panel. The panel shown in Fig. 1.5 has size $L_1 \times L_2$. A panel may be divided into column strips and middle strips. Column Strip means a design strip having a width of $0.25L_1$ or $0.25L_2$, whichever is less. The remaining middle portion which is bound by the column strips is called middle strip. Fig. 1.5 shows the division of flat slab panel into column and middle strips in the direction y.

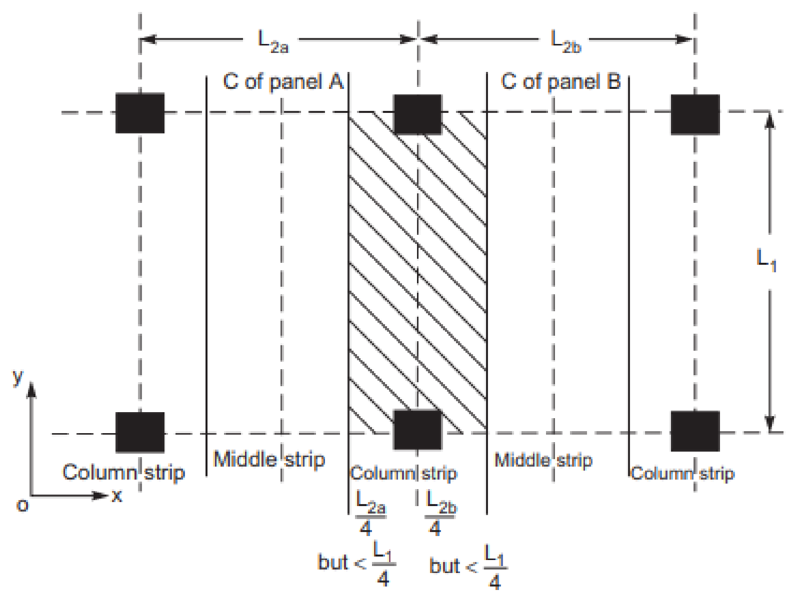


Fig. 1.5 *Panels, column strips and middle strips is y-direction*

Proportioning of Flat Slabs

IS 456-2000 [Clause 31.2] gives the following guidelines for proportioning:

- Drops: The drops when provided shall be rectangular in plan, and have a length in each direction not less than one third of the panel in that direction. For exterior panels, the width of drops at right angles to the non-continuous edge and measured from the centre-line of the columns shall be equal to one half of the width of drop for interior panels.
- Column Heads: Where column heads are provided, that portion of the column head which lies within the largest right circular cone or pyramid entirely within the outlines of the column and the column head, shall be considered for design purpose as shown in Figs. 1.2 and 1.4.
- Thickness of Flat Slabs: From the consideration of deflection control IS 456-2000 specifies minimum thickness in terms of span to effective depth ratio. For this purpose larger span is to be considered. If drop as specified in 1.2.1 is provided, then the maximum value of ratio of larger span to thickness shall be
  = 40, if mild steel is used
  = 32, if Fe 415 or Fe 500 steel is used
  If drops are not provided or size of drops do not satisfy the specification 1.2.1, then the ratio shall not exceed 0.9 times the value specified above i.e.,
  = 40 × 0.9 = 36, if mild steel is used.
  = 32 × 0.9 = 28.8, if HYSD bars are used
  It is also specified that in no case, the thickness of flat slab shall be less than 125 mm.

For Determination of Bending Moment & Shear Force:

For this IS 456-2000 permits use of any one of the following two methods:

a) The Direct Design Method
b) The Equivalent Frame Method

THE DIRECT DESIGN METHOD

This method has the limitation that it can be used only if the following conditions are fulfilled:

i. There shall be minimum of three continuous spans in each directions.
ii. The panels shall be rectangular and the ratio of the longer span to the shorter span within a panel shall not be greater than 2.
iii. The successive span length in each direction shall not differ by more than one-third of longer span.
iv. The design live load shall not exceed three times the design dead load.
v. The end span must be shorter but not greater than the interior span.
vi. It shall be permissible to offset columns a maximum.

Total Design Moment

The absolute sum of the positive and negative moment in each direction is given by

$$M_0 = \frac{WL_n}{8}$$

Where,

M_0 = Total moment

W = Design load on the area $L_2 \times L_n$

L_n = Clear span extending from face to face of columns, capitals, brackets or walls but not less than 0.65 L_1

L_1 = Length of span in the direction of M_0; and

L_2 = Length of span transverse to L_1

In taking the values of L_n, L_1 and L_2, the following clauses are to be carefully noted:

(*a*) Circular supports shall be treated as square supports having the same area *i.e.*, squares of size 0.886D.

(*b*) When the transverse span of the panel on either side of the centre line of support varies, L_2 shall be taken as the average of the transverse spans. In Fig. 1.5 it is given by $\frac{(L_{2a} + L_{2b})}{2}$.

(*c*) When the span adjacent and parallel to an edge is being considered, the distance from the edge to the centre-line of the panel shall be substituted for L_2.

Distribution of Bending Moment in to -ve and +ve Moments

The total design moment M0 in a panel is to be distributed into –ve moment and +ve moment as specified below:

In an interior span

Negative Design Moment	0.65 M_0
Positive Design Moment	0.35 M_0

In an end span

Interior negative design moment

$$= \left[0.75 - \frac{0.10}{1 + \frac{1}{\alpha_c}}\right] M_0$$

Positive design moment

$$= \left[0.63 - \frac{0.28}{1 + \frac{1}{\alpha_c}}\right] M_0$$

Exterior negative design moment

$$= \left[\frac{0.65}{1+\frac{1}{\alpha_c}}\right] M_0$$

where α_c is the ratio of flexural stiffness at the exterior columns to the flexural stiffness of the slab at a joint taken in the direction moments are being determined and is given by

$$\alpha_c = \frac{\sum K_c}{\sum K_s}$$

Where,

K_c = Sum of the flexural stiffness of the columns meeting at the joint; and

K_s = Flexural stiffness of the slab, expressed as moment per unit rotation.

Distribution of the Bending Moment across the Panel Width

The +ve and –ve moments found are to be distributed across the column strip in a panel as shown in Table 1.1. The moment in the middle strip shall be the difference between panel and the column strip moments.

Table 1.1 Distribution of Moments Across the Panel Width in a Column Strip

S. No.	*Distributed Moment*	*Per cent of Total Moment*
a	Negative BM at the exterior support	100
b	Negative BM at the interior support	75
c	Positive bending moment	60

Moments in Columns

In this type of constructions column moments are to be modified as suggested in IS 456–2000 [Clause No. 31.4.5].

Shear Force

The critical section for shear shall be at a distance d/2 from the periphery of the column/capital drop panel. Hence if drops are provided there are two critical sections near columns. These critical sections are shown in Figs. 1.1 to 1.4. The shape of the critical section in plan is similar to the support immediately below the slab as shown in Fig. 1.6.

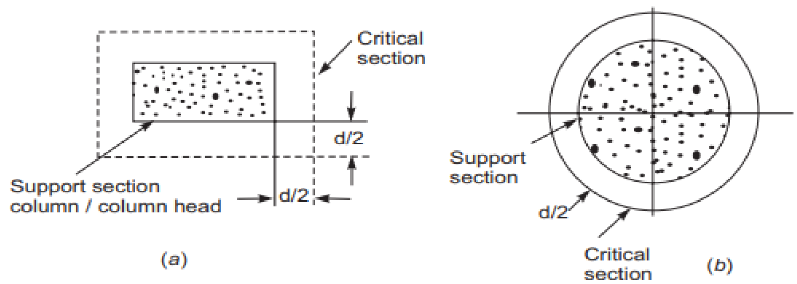


Fig. 1.6

For columns sections with re-entrant angles, the critical section shall be taken as indicated in Fig. 1.7.

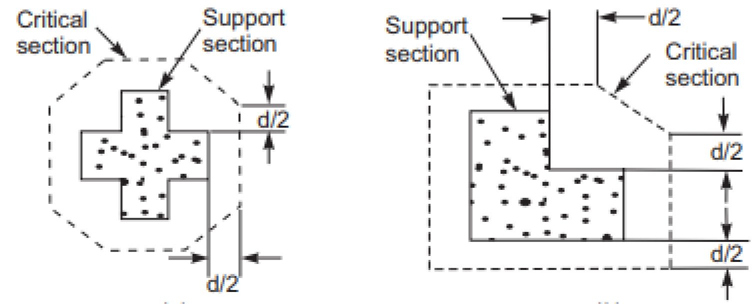


Fig. 1.7

In case of columns near the free edge of a slab, the critical section shall be taken as shown in Fig. 1.8.

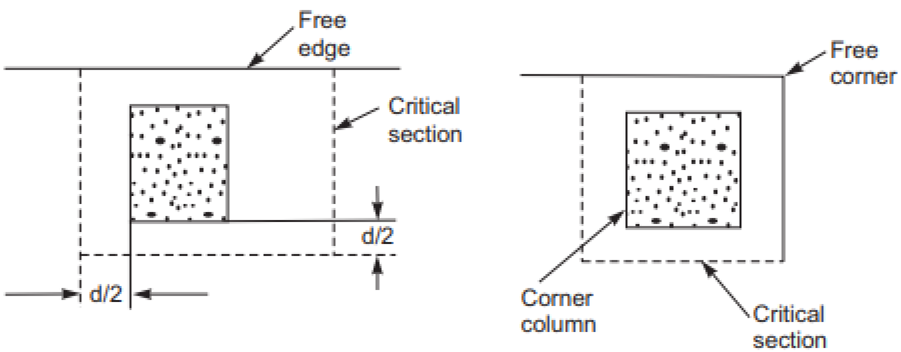


Fig. 1.8

The nominal shear stress may be calculated as

$$\tau_v = \frac{V}{b_0 d}$$

where V – is shear force due to design

b_0 – is the periphery of the critical section

d – is the effective depth

The permissible shear stress in concrete may be calculated as $k_s\,\tau_c$, where $k_s = 0.5 + \beta_c$ but not greater than 1, where β_c is the ratio of short side to long side of the column/capital; and

$$\tau_c = 0.25\sqrt{f_{ck}}$$

If shear stress $\tau_v < \tau_c$ – no shear reinforcement are required. If $\tau_c < \tau_v < 1.5\,\tau_c$, shear reinforcement shall be provided. If shear stress exceeds 1.5 τ_c flat slab shall be redesigned.

EQUIVALENT FRAME METHOD

IS 456–2000 recommends the analysis of flat slab and column structure as a rigid frame to get design moment and shear forces with the following assumptions:

a. Beam portion of frame is taken as equivalent to the moment of inertia of flat slab bounded laterally by centre line of the panel on each side of the centre line of the column. In frames adjacent and parallel to an edge beam portion shall be equal to flat slab bounded by the edge and the centre line of the adjacent panel.
b. Moment of inertia of the members of the frame may be taken as that of the gross section of the concrete alone.
c. Variation of moment of inertia along the axis of the slab on account of provision of drops shall be taken into account. In the case of recessed or coffered slab which is made solid in the region of the columns, the stiffening effect may be ignored provided the solid part of the slab does not extend more than 0.15 lef into the span measured from the centre line of the columns. The stiffening effect of flared columns heads may be ignored.
d. Analysis of frame may be carried out with substitute frame method or any other accepted method like moment distribution or matrix method.

Loading Pattern

When the live load does not exceed ¾th of dead load, the maximum moments may be assumed to occur at all sections when full design live load is on the entire slab. If live load exceeds ¾th dead load analysis is to be carried out for the following pattern of loading also:

i. To get maximum moment near mid span – ¾th of live load on the panel and full live load on alternate panel
ii. To get maximum moment in the slab near the support – ¾th of live load is on the adjacent panel only

It is to be carefully noted that in no case design moment shall be taken to be less than those occurring with full design live load on all panels.

The moments determined in the beam of frame (flat slab) may be reduced in such proportion that the numerical sum of positive and average negative moments is not less than the value of total design moment
$M_0 = WL_n / 8$.

The distribution of slab moments into column strips and middle strips is to be made in the same manner as specified in direct design method.

Slab Reinforcement

- Spacing: The spacing of bars in a flat slab, shall not exceed 2 times the slab thickness.
- Area of Reinforcement: When the drop panels are used, the thickness of drop panel for determining area of reinforcement shall be the lesser of the following:
  - Thickness of drop, and
  - Thickness of slab plus one quarter the distance between edge of drop and edge of capital.
  - The minimum percentage of the reinforcement is same as that in solid slab i.e., 0.12 percent if HYSD bars used and 0.15 percent, if mild steel is used.

- Minimum length of Reinforcement: At least 50 percent of bottom bars should be from support to support. The rest may be bent up. The minimum length of different reinforcement in flat slabs should be as shown in Fig. 1.9 (Fig. 16 in IS 456– 2000). If adjacent spans are not equal, the extension of the –ve reinforcement beyond each face shall be based on the longer span. All slab reinforcement should be anchored property at discontinuous edges.

NUMERICALS

Example 1.1: *Design an interior panel of a flat slab of size 5 m × 5 m without providing drop and column head. Size of columns is 500 × 500 mm and live load on the panel is 4 kN/m². Take floor finishing load as 1 kN/m². Use M20 concrete and Fe 415 steel.*

Solution:

Thickness

Since drop is not provided and HYSD bars are used span to thickness ratio shall not exceed

$$\frac{1}{0.9 \times 32} = \frac{1}{28.8}$$

∴ Minimum thickness required

$$= \frac{\text{Span}}{28.8} = \frac{5000}{28.8} = 173.6 \text{ mm}$$

Let $d = 175$ mm and D = 200 mm

Loads

Self weight of slab $= 0.20 \times 25 = 5 \text{ kN/m}^2$

Finishing load $= 1 \text{ kN/m}^2$

Live load $= 4 \text{ kN/m}^2$

∴ Total working load $= 10 \text{ kN/m}^2$

Factored load $= 1.5 \times 10 = 15 \text{ kN/m}^2$

$$L_n = 5 - 0.5 = 4.5 \text{ m}$$

∴ Total design load in a panel $W = 15\, L_2 L_n = 15 \times 5 \times 4.5 = 337.5$ kN

Moments

Panel Moment $M_0 = \frac{WL_n}{8} = 337.5 \times \frac{4.5}{8} = 189.84$ kNm

Panel –ve moment $= 0.65 \times 189.84 = 123.40$ kNm

Panel +ve moment $= 0.35 \times 189.84 = 0.35 \times 189.84 = 66.44$ kNm

Distribution of moment into column strips and middle strip:

	Column Strip in kNm	*Middle Strip in kNm*
–ve moment	$0.75 \times 123.40 = 92.55$	30.85
+ve moment	$0.60 \times 66.44 = 39.86$	26.58

Checking the thickness selected:

Since Fe 415 steel is used,

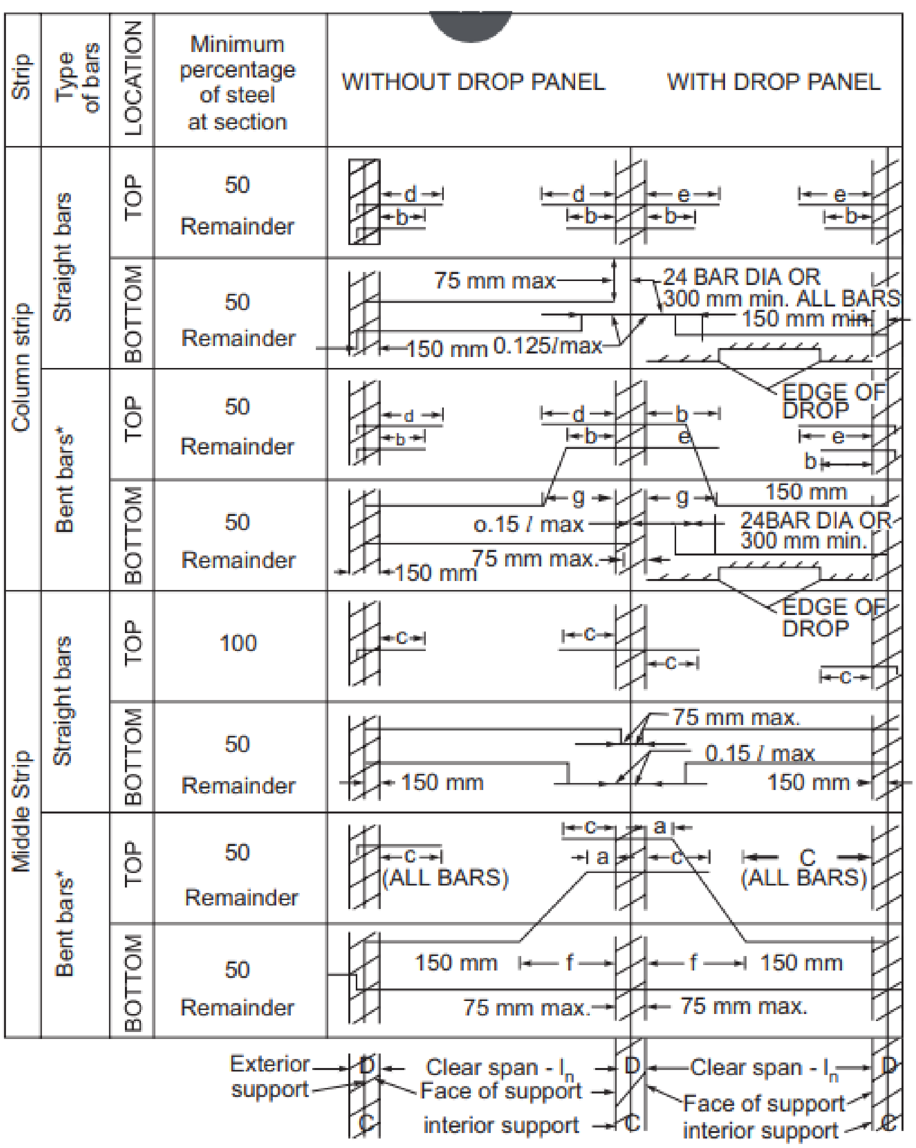
$$M_{u\,lim} = 0.138\, f_{ck}\, b\, d^2$$

Width of column strip $= 0.5 \times 5000 = 2500$ mm

$$\therefore \quad M_{u\,lim} = 0.138 \times 20 \times 2500 \times 175^2 = 211.3125 \times 10^6 \text{ Nmm}$$

$$= 211.3125 \text{ kNm}$$

Hence singly reinforced section can be designed *i.e.*, thickness provided is satisfactory from the consideration of bending moment.



[NO SLAB CONTINUITY] [CONTINUITY PROVED] [NO SLAB CONTINUITY]

Bar Length From Face of Support							
	Minimum Length					Maximum Length	
Mark	a	b	c	d	e	f	g
Length	0.14 l_n	0.20 l_n	0.22 l_n	0.30 l_n	0.33 l_n	0.20 l_n	0.24 l_n

* Bent bars at exterior supports may be used if a general analysis is made.

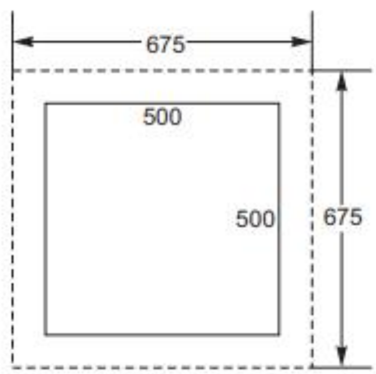
Note. D is the diameter of the column and the dimension of the rectangular column in the direction under consideration.

Fig. 1.9 *Minimum bend joint locations and extensions for reinforcement in flat slabs*

Check for Shear

The critical section for shear is at a distance $\frac{d}{2}$ from the column face. Hence periphery of critical section around a column is square of a size = 500 + d = 500 + 175 = 675 mm

Shear to be resisted by the critical section



$$V = 15 \times 5 \times 5 - 15 \times 0.675 \times 0.675$$
$$= 368.166 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{368.166 \times 1000}{4 \times 675 \times 175} = 0.779 \text{ N/mm}^2$$

$k_s = 1 + \beta_c$ subject to maximum of 1.

$$\beta_c = \frac{L_1}{L_2} = \frac{5}{5} = 1$$

$$\therefore \quad k_s = 1$$

$$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118 \text{ N/mm}^2$$

safe in shear since $\tau_v < \tau_c$

Reinforcement

For –ve moment in column strip:

$$M_u = 92.55 \text{ kNm}$$

$$92.55 \times 10^6 = 0.87 f_y\, A_{st}\, d\left[1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right]$$

$$= 0.87 \times 415 \times A_{st} \times 175\left[1 - \frac{A_{st}}{2500 \times 175} \times \frac{415}{20}\right]$$

i.e., $$1464.78 = A_{st}\left[1 - \frac{A_{st}}{21084.3}\right]$$

i.e., $$A_{st}^2 - 21084.3A_{st} + 1464.78 \times 21084.3 = 0$$

$$A_{st} = 1583.74 \text{ mm}^2$$

This is to be provided in a column strip of width 2500 mm. Hence using 12 mm bars, spacing required is given by

$$s = \frac{\pi/4 \times 12^2}{1583.74} \times 2500 = 178 \text{ mm}$$

Provide 12 mm bars at 175 mm c/c.

For +ve moment in column strip:

$$M_u = 39.86 \text{ kNm}$$

$$\therefore \quad 39.86 \times 10^6 = 0.87 \times 415 \times A_{st} \times 175\left[1 - \frac{A_{st}}{2500 \times 175} \times \frac{415}{20}\right]$$

$$630.86 = A_{st}\left[1 - \frac{A_{st}}{21084.3}\right]$$

or $$A_{st}^2 - 21084.3\ A_{st} + 630.86 \times 21084.3 = 0$$

$$\therefore \quad A_{st} = 651 \text{ mm}^2$$

Using 10 mm bars, spacing required is

$$s = \frac{\pi/4 \times 10^2}{651} \times 2500 = 301.6 \text{ mm} < 2 \times \text{thickness of slab}$$

Hence provide 10 mm bars at 300 mm c/c.

Provide 10 mm diameter bars at 300 mm c/c in the middle strip to take up –ve and +ve moments. Since span is same in both directions, provide similar reinforcement in other direction also.

Reinforcement Details

It is as shown in Fig. 1.10

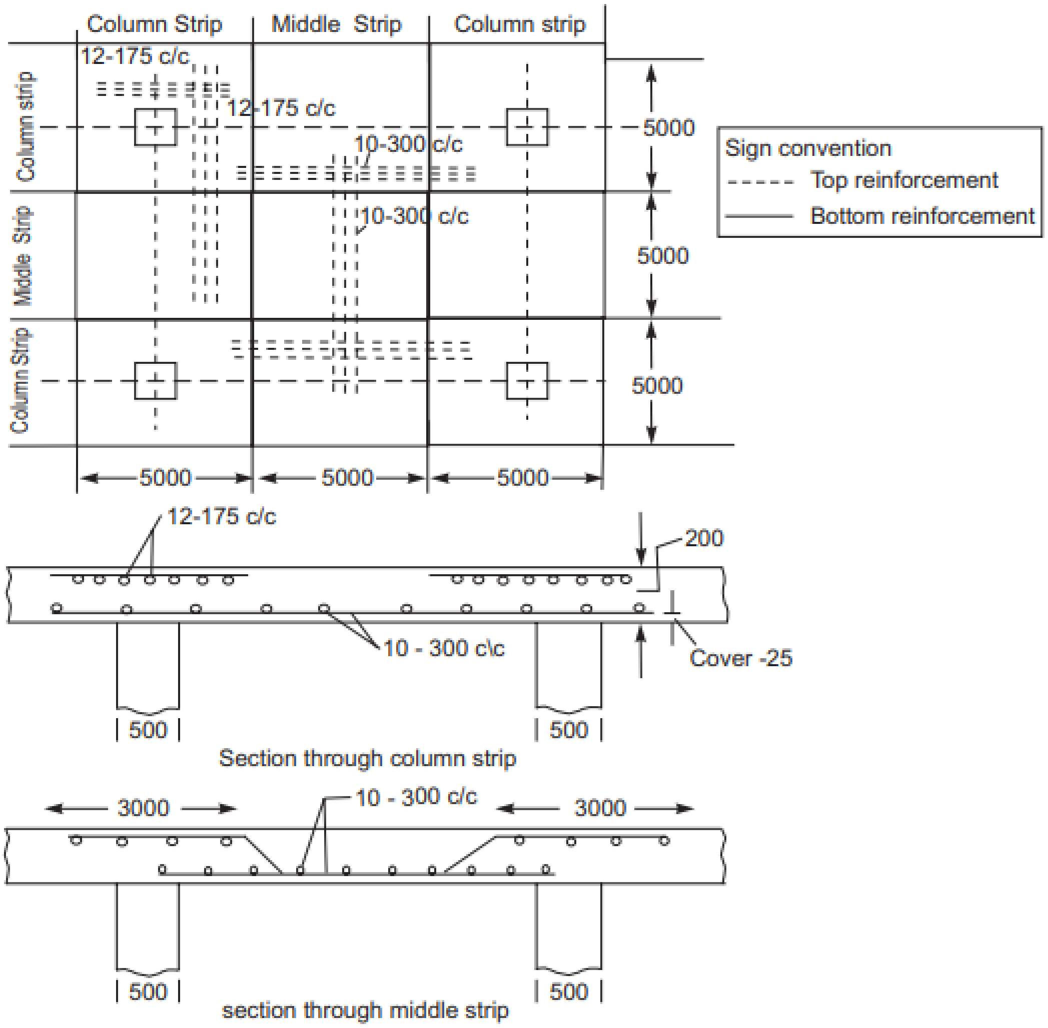


Fig. 1.10 *Reinforcement details [all dimension in mm units]*

Example 1.2: *Design an interior panel of a flat slab with panel size 6 × 6 m supported by columns of size 500 × 500 mm. Provide suitable drop. Take live load as 4 kN/m². Use M20 concrete and Fe 415 steel.*

Solution :

Thickness : Since Fe 415 steel is used and drop is provided, maximum span to thickness ratio permitted is 32

$$\therefore \text{ Thickness of flat slab } = \frac{6000}{32} = 187.5 \text{ mm}$$

Provide 190 mm thickness. Let the cover be 30 mm

∴ Overall thickness D = 220 mm

Let the drop be 50 mm. Hence at column head, d = 240 mm and D = 270 mm

Size of Drop

It should not be less than $\frac{1}{3} \times 6 \text{ m} = 2 \text{ m}$

Let us provide 3 m × 3 m drop so that the width of drop is equal to that of column head.

∴ Width of column strip = width of middle strip = 3000 mm.

Loads

For the purpose of design let us take self-weight as that due to thickness at column strip

∴ Self-weight $= 0.27 \times 1 \times 1 \times 25 = 6.75 \text{ kN/m}^2$

Finishing load $= 1.00 \text{ kN/m}^2$

Live load $= 4.00 \text{ kN/m}^2$

Total load $= 11.75 \text{ kN/m}^2$

∴ Design (factored) load $= 1.5 \times 11.75 = 17.625 \text{ kN/m}^2$

Clear span $L_n = 6 - 0.5 = 5.5 \text{ m}$

∴ Design load $W_0 = W_u \times L_2 \times L_n$

$= 17.625 \times 6 \times 5.5$

$= 581.625 \text{ kN}$

Design Total Moment

Total moment

$$M_0 = \frac{W_0 L_n}{8} = \frac{581.625 \times 5.5}{8} = 400 \text{ kNm}$$

∴ Total negative moment $= 0.65 \times 400 = 260 \text{ kNm}$

Total positive moment $= 0.35 \times 400 = 140 \text{ kNm}$

The above moments are to be distributed into column strip and middle strip

	Column Strip	*Middle Strip*
–ve moment	$0.75 \times 260 = 195$ kNm	$0.25 \times 260 = 65$ kNm
+ve moment	$0.6 \times 140 = 84$ kNm	$0.4 \times 140 = 56$ kNm

Width of column strip = width of middle strip = 3000 mm

$M_{u\,lim} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 3000 \times 240^2 = 476.928 \times 10^6 \text{ Nmm}$

$= 476.928 \text{ kNm}$

Thus $M_{u\,lim} > M_u$. Hence thickness selected is sufficient.

Check for Shear

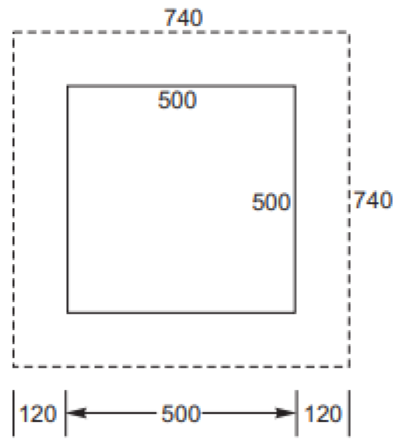
The critical section is at a distance

$$\frac{d}{2} = \frac{240}{2} = 120 \text{ mm from the face of column}$$

$\therefore$ It is a square of size $= 500 + 240 = 740$ mm

$$V = \text{Total load} - \text{load on } 0.740 \times 0.740 \text{ area}$$
$$= 17.625 \times 6 \times 6 - 17.625 \times 0.740 \times 0.740$$
$$= 624.849 \text{ kN}$$

$\therefore$ Nominal shear $= \tau_v = \dfrac{624.489 \times 1000}{4 \times 740 \times 240} = 0.880 \text{ N/mm}^2$



Shear strength $= k_s \, \tau_c$

where $k_s = 1 + \beta_c$ subject to maximum of 1

where $\beta_c = \dfrac{L_1}{L_2} = 1$

$\therefore$ $k_s = 1$

$$\tau_c = 0.25\sqrt{20} = 1.118 \text{ N/mm}^2$$

Design shear stress permitted

$$= 1.118 \text{ N/mm}^2 > \tau_v$$

Hence the slab is safe in shear without shear reinforcement also.

Shear strength may be checked at distance $\dfrac{d}{2}$ from drop. It is quite safe since drop size is large.

Reinforcement

(*a*) For –ve moment in column strip

$$M_u = 195 \text{ kNm}$$

Thickness $d = 240$ mm

$\therefore$
$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st}}{b \times d} \times \frac{f_y}{f_{ck}}\right]$$

$$195 \times 10^6 = 0.87 \times 415 \times A_{st} \times 240 \left[1 - \frac{A_{st}}{3000 \times 240} \times \frac{415}{20}\right]$$

$$2250.38 = A_{st}\left[1 - \frac{A_{st}}{34698.8}\right]$$

$$A_{st}^2 - 34698.8\ A_{st} + 2250.38 \times 34698.8 = 0$$

$$A_{st} = 2419 \text{ mm}^2 \text{ in } 3000 \text{ mm width}$$

Using 12 mm bars, spacing required is

$$s = \frac{\pi/4 \times 12^2}{2419} \times 3000 = 140.26 \text{ mm}$$

Provide 12 mm bars at 140 mm c/c

(*b*) For +ve moment in column strip

$$M_u = 84 \text{ kNm} = 84 \times 10^6 \text{ Nmm. Thickness } d = 190 \text{ mm}$$

$$84 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190 \left[1 - \frac{A_{st}}{3000 \times 240} \times \frac{415}{20}\right]$$

$$1224.5 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$\therefore \qquad A_{st} = 1285 \text{ mm}^2$$

Using 10 mm bars

$$s = \frac{\pi/4 \times 10^2}{1285} \times 3000 = 183 \text{ mm}$$

Provide 10 mm bars at 180 mm c/c

(*c*) For –ve moment in middle strip:

$$M_u = 65 \text{ kNm}; \qquad \text{Thickness} = 190 \text{ mm}$$

$$65 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190 \left[1 - \frac{A_{st}}{3000 \times 190} \times \frac{415}{20}\right]$$

$$947.5 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$A_{st}^2 - 27469.9\ A_{st} + 947.5 \times 27469.9 = 0$$

$$A_{st} = 983 \text{ mm}^2 \text{ in } 3000 \text{ mm width}$$

Using 10 mm bars

$$s = \frac{\pi/4 \times 10^2}{983} \times 3000 = 239.7 \text{ mm}$$

Provide 10 mm bars at 230 mm c/c

(*d*) For +ve moment in middle strip

$$M_u = 56 \text{ kNm}; \qquad \text{Thickness} = 190 \text{ mm}$$

Provide 10 mm bars at 230 mm c/c in this portion also.

Since span is same in both direction, provide similar reinforcement in both directions. The details of reinforcement are shown in Fig. 1.11.

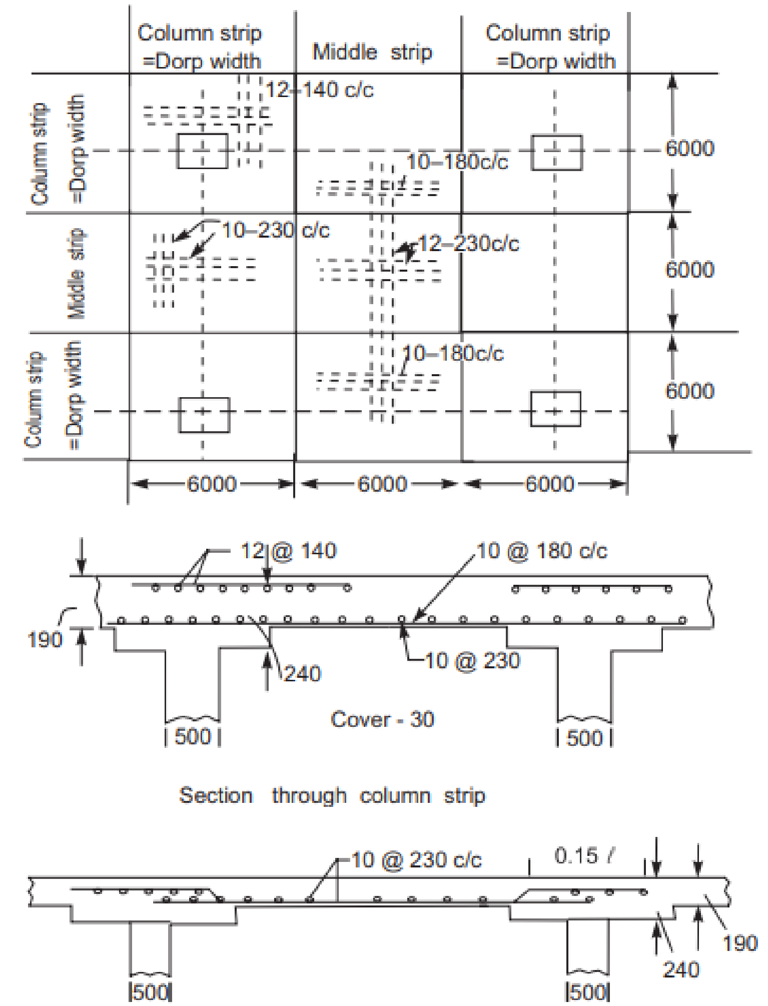


Fig. 1.11 *Reinforcement details*

Example 1.3: *Design the interior panel of the flat slab in example 1.2, providing a suitable column head, if columns are of 500 mm diameter.*

Solution: Let the diameter of column head be

$$= 0.25L = 0.25 \times 6 = 1.5 \text{ m}$$

It's equivalent square has side '*a*' where

$$\frac{\pi}{4} \times 1.5^2 = a^2$$

$$a = 1.33 \text{ m}$$

$$\therefore \quad L_n = 6 - 1.33 = 4.67 \text{ m}$$

$$W_0 = 17.625 \times 6 \times 4.67 = 493.85 \text{ kN}$$

$$M_0 = \frac{W_o L_n}{8} = \frac{493.85 \times 4.67}{8} = 288.3 \text{ kNm}$$

$\therefore$ Total –ve moment = 0.65 × 288.3 = 187.4 kNm

Total +ve moment = 0.35 × 288.3 = 100.9 kNm

The distribution of above moment into column strip and middle strips are as given below:

	Column Strip	*Middle Strip*
–ve moment	$0.75 \times 187.4 = 140.55$ kNm	$0.25 \times 187.4 = 46.85$ kNm
+ve moment	$0.60 \times 100.9 = 60.54$ kNm	$0.4 \times 100.9 = 40.36$ kNm

Width of column strip = width of middle strip = 3000 mm

$$\therefore \quad M_{u\,lim} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 3000 \times 240^2$$

$$= 476.928 \times 10^6 \text{ Nmm} > M_u$$

Hence thickness selected is sufficient.

Check for Shear

The critical section is at a distance

$$\frac{d}{2} = \frac{240}{2} = 120 \text{ mm from the face of column head}$$

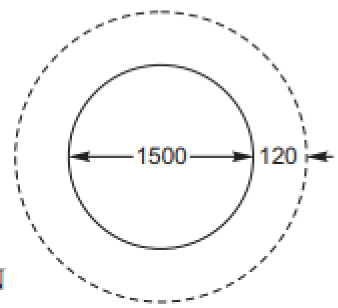
Diameter of critical section = 1500 + 240 = 1740 mm

= 1.740 m

Perimeter of critical section = π D

= 1.740 π

Shear on this section



$$V = 17.625\left[6 \times 6 - \frac{\pi}{4} \times 1.74^2\right] = 592.59 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{592.59 \times 1000}{\pi \times 1740 \times 240} = 0.45 \text{ N/mm}^2$$

Maximum shear permitted = $k_s \times 0.25\sqrt{20}$

= 1.118 N/mm² Since k_s works out to be 1

Since maximum shear permitted in concrete is more than nominal shear τ_v, there is no need to provide shear reinforcement

Design of Reinforcement

(*a*) For –ve moment in column strip

$$M_u = 140.55 \text{ kNm}; \quad d = 240 \text{ mm}$$

$$\therefore \quad 140.55 \times 10^6 = 0.87 \times 415 \times A_{st} \times 240\left[1 - \frac{A_{st}}{3000 \times 240} \times \frac{415}{20}\right]$$

$$1622 = A_{st}\left[1 - \frac{A_{st}}{34698.8}\right]$$

$$A_{st}^2 - 34698.8\ A_{st} + 1622 \times 34698.8 = 0$$

$$A_{st} = 1705\ \text{mm}^2$$

Using 12 mm bars,

$$s = \frac{\pi/4 \times 12^2}{1705} \times 3000 = 199\ \text{mm}$$

Provide 12 mm bars at 190 mm c/c.

(*b*) For the +ve moment in column strip

$$M_u = 60.54\ \text{kNm}; \qquad d = 190\ \text{mm}$$

$$60.54 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190 \left[1 - \frac{A_{st}}{3000 \times 190} \times \frac{415}{20}\right]$$

$$882.51 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$A_{st}^2 - 27469.9\ A_{st} + 882.51 \times 27469.9 = 0$$

$$A_{st} = 913\ \text{mm}^2$$

Using 10 mm bars

$$s = \frac{\pi/4 \times 10^2}{913} \times 3000 = 258\ \text{mm}$$

Provide 10 mm bars at 250 mm c/c.

(*c*) For –ve moment in middle strip:

$$M_u = 46.85\ \text{kNm}; \qquad d = 190\ \text{mm}$$

$$46.85 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190\left[1 - \frac{A_{st}}{3000 \times 190} \times \frac{415}{20}\right]$$

$$683 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$A_{st}^2 - 27469.9 A_{st} + 683 \times 27469.9 = 0$$

$$A_{st} = 701\ \text{mm}^2$$

Using 10 mm bars,

$$s = \frac{\pi/4 \times 10^2}{701} \times 3000 = 336\ \text{mm}$$

Provide 10 mm bars at 300 mm c/c.

(*d*) Provide 10 mm bars at 300 mm c/c for +ve moment in middle strip also.

As span is same in both directions, provide similar reinforcement in both directions. Reinforcement detail may be shown as was done in previous problem.

Example 1.4: *A flat slab system consists of 5 m × 6 m panels and is without drop and column head. It has to carry a live load of 4 kN/m² and a finishing load of 1 kN/m². It is to be designed using M20 grade concrete and Fe 415 steel. The size of the columns supporting the system is 500 × 500 mm and floor to floor height is 4.5 m. Calculate design moments in interior and exterior panels at column and middle strips in both directions.*

Solution:

Thickness: Since Fe 415 steel is used and no drops are provided, longer span to depth ratio is not more than $32 \times 0.9 = 28.8$

$$d = \frac{6000}{28.8} = 208$$

Let us select $d = 210$ mm and D = 240 mm

Loads

Self weight	$0.24 \times 1 \times 1 \times 25 = 6$ kN/m^2
Finishing weight	$= 1$ kN/m^2
Live load	$= 4$ kN/m^2
Total	$= 11$ kN/m^2

$$W_u = 1.5 \times 11 = 16.5 \text{ kN/m}^2$$

Panel Dimensions

Along length

$L_1 = 6$ m and $L_2 = 5$ m

Width of column strip $= 0.25\ L_1$ or L_2 whichever is less.

$= 0.25 \times 5 = 1.25$ m on either side of column centre line

$\therefore$ Total width of column strip $= 1.25 \times 2 = 2.5$ m

Width of middle strip $= 5 - 2.5 = 2.5$ m

Along Width

$L_1 = 5$ m $\quad L_2 = 6$ m

Width of column strip $= 0.25 \times 5 = 1.25$ m on either side

$\therefore$ Total width of column strip $= 2.5$ m

Hence, width of middle strip $= 6 - 2.5 = 3.5$ m

INTERIOR PANELS

Moments Along Longer Size

$L_1 = 6$ m $\quad L_2 = 5$ m

$L_n = 6 - 0.5 = 5.5$ m subject to minimum of $0.65 \times L_1 = 3.9$ m

$\therefore$ $L_n = 5.5$ m

Load on panel $W_0 = 16.5 \times L_2 L_n$

$= 16.5 \times 5 \times 5.5 = 453.75$ kN

$$M_0 = \frac{W_0\, L_n}{8} = \frac{453.75 \times 5.5}{8} = 311.95 \text{ kNm}$$

Appropriation of Moment

Total –ve moment = 0.65 × 311.95 = 202.77 kNm

∴ Total +ve moment = 311.95 – 202.77 = 109.18 kNm

Hence moment in column strip and middle strip along longer direction in interior panels are as given below:

	Column Strip	*Middle Strip*
–ve moment	$0.75 \times 202.75 = 152.06$ kNm	$202.75 - 152.06 = 50.69$ kNm
+ve moment	$0.60 \times 109.18 = 65.51$ kNm	$109.18 - 65.51 = 43.67$ kNm

Along Width

$$L_1 = 5 \text{ m} \quad L_2 = 6 \text{ m} \quad \text{and} \quad L_n = 5 - 0.5 = 4.5 \text{ m.}$$

Panel load $= W_0 = 16.5 \times 6 \times 4.5 = 445.5$ kN

Panel moment $M_0 = W_0 \dfrac{L_n}{8} = \dfrac{445.5 \times 4.5}{8} = 250.59$ kN-m

Appropriation of Moment:

Total –ve moment $= 0.65 \times 250.59 = 162.88$ kN-m

Total +ve moment $= 250.59 - 162.88 = 87.71$ kN-m

∴ Moments in column strip and middle strip are as shown below:

	Column Strip	*Middle Strip*
–ve moment	$0.75 \times 162.88 = 122.16$ kNm	$0.25 \times 162.88 = 40.72$ kNm
+ve moment	$0.60 \times 87.71 = 52.63$ kNm	$0.40 \times 87.71 = 35.08$ kNm

EXTERIOR PANELS

Length of column $= 4.5 - 0.24 = 4.26$ m

The building is not restrained from lateral sway. Hence as per Table 28 in IS 456-2000, effective length of column

$$= 1.2 \times \text{length} = 1.2 \times 4.26 = 5.112 \text{ m}$$

Size of column $= 500 \times 500$ mm

Moment of inertia of column $= \dfrac{1}{12} \times 500^4 \text{ mm}^4$

$$\therefore \quad k_c = \frac{I}{L} = \frac{1}{12} \times \frac{500^4}{5112} = 101844 \text{ mm}^4$$

LONGER SPAN DIRECTION

Moment of inertia of beam

I_s = Moment of inertia of slab

$$= \frac{1}{12} \times 6000 \times 240^3$$

Its length $= L_2 = 5000$ mm

$$\therefore \quad k_c = \frac{I_s}{5000} = \frac{1}{12} \times \frac{6000 \times 240^3}{5000} = 1382400 \text{ mm}^4$$

$$\frac{\text{Live load}}{\text{Dead load}} = \frac{4}{7} < 0.75$$

$\therefore$ Relative stiffness ratio is

$$\alpha_c = \frac{k_{c1} + k_{c2}}{k_s} = \frac{2 \times 1018844}{1382400} = 1.474$$

$$\therefore \quad \alpha = 1 + \frac{1}{\alpha_c} = 1 + \frac{1}{1.474} = 1.678$$

Hence various moment coefficients are:

Interior –ve moment coefficient $= 0.75 - \frac{0.1}{\alpha} = 0.690$

Exterior –ve moment coefficient $= \frac{0.65}{\alpha} = 0.387$

Positive moment coefficient $= 0.63 - \frac{0.28}{\alpha} = 0.463$

Total moment $M_0 = 311.95$ kNm

$\therefore$ Appropriation of moments in kNm is as given below:

	Total	*Column Strip*	*Middle Strip*
Interior –ve	$0.69 \times 311.95 = 215.25$	$0.75 \times 215.25 = 161.43$	$215.25 - 161.43 = 53.82$
Exterior –ve	$0.387 \times 311.95 = 120.72$	$1.00 \times 120.72 = 120.72$	$120.72 - 120.72 = 0$
+ Moment	$0.463 \times 31.95 = 144.43$	$0.60 \times 144.43 = 86.66$	$144.43 - 86.66 = 57.77$

Shorter Span Direction

$$\therefore \quad k_s = \frac{1}{12} \times \frac{5000 \times 240^3}{6000} = 96000$$

$$\therefore \quad \alpha_c = \frac{k_{c1} + k_{c2}}{k_s} = \frac{2 \times 1018844}{960000} = 2.123$$

$$\therefore \quad \alpha_1 = 1 + \frac{1}{\alpha_c} = 1.471$$

Interior –ve moment coefficient $= 0.75 - \dfrac{0.1}{\alpha} = 0.75 - \dfrac{0.1}{1.471} = 0.682$

Exterior –ve moment coefficient $= \dfrac{0.65}{\alpha} = \dfrac{0.65}{1.471} = 0.442$

Positive moment coefficient $= 0.63 - \dfrac{0.28}{\alpha} = 0.63 - \dfrac{0.28}{1.471} = 0.440$

Total moment $M_0 = 250.59$ kNm

∴ Appropriation of moments in shorter span exterior panel in kNm is as given below:

	Total	*Column Strip*	*Middle Strip*
Interior –ve	$0.682 \times 250.59 = 170.90$	$0.75 \times 170.76 = 128.18$	$170.90 - 128.18 = 42.72$
Exterior -ve	$0.442 \times 250.59 = 110.76$	$1.00 \times 110.76 = 110.76$	$110.76 - 110.76 = 0$
+ Moment	$0.44 \times 250.59 = 110.25$	$0.60 \times 110.25 = 66.16$	$110.25 - 66.16 = 44.09$

In the exterior panel in each column strips half the above values will act. These moments are shown in Fig. 1.12

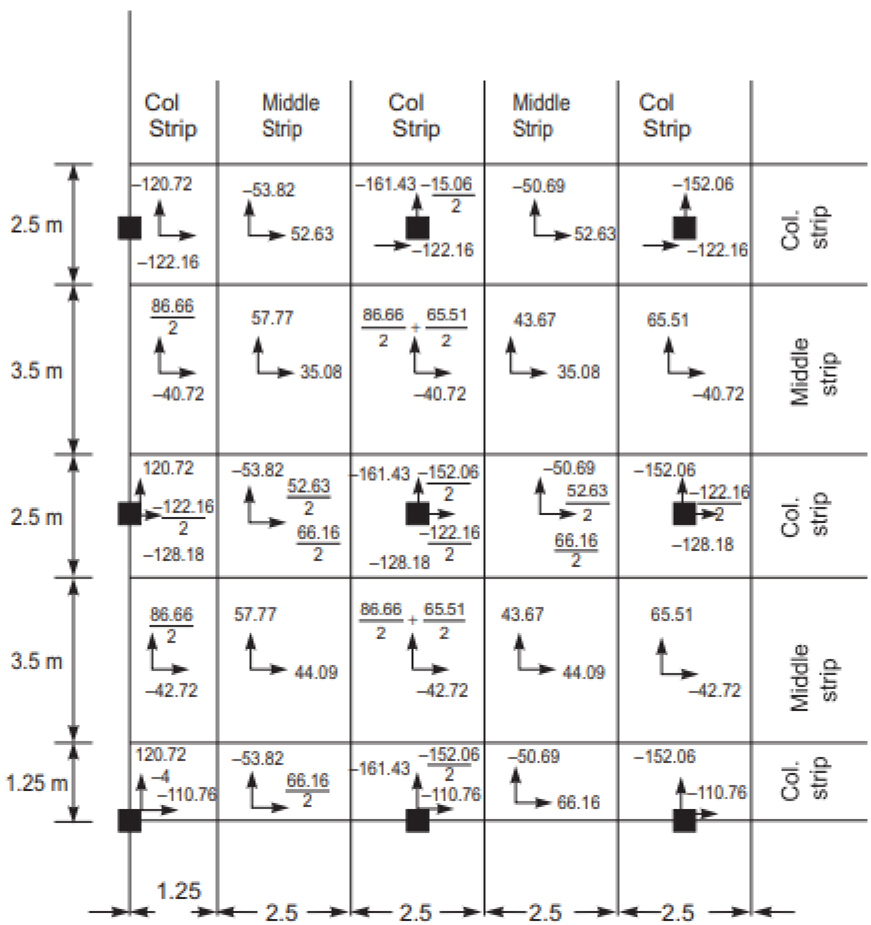


Fig. 1.12

RETAINING WALL ANALYSIS & DESIGN

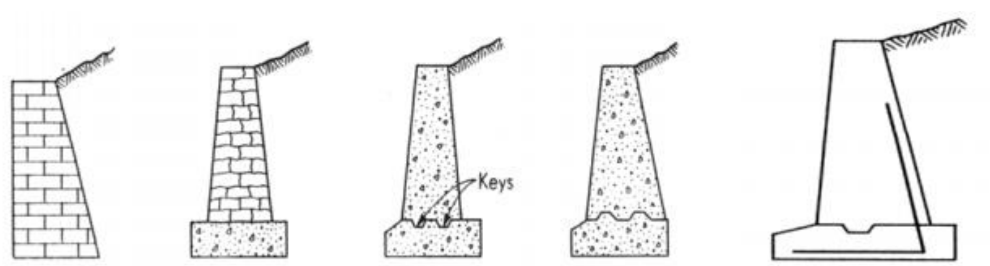
INTRODUCTION

Retaining walls are structures used to provide stability for earth or other materials at their natural slopes. In general, they are used to hold back or support soil banks and water or to maintain difference in the elevation of the ground surface on each of wall sides. Also, retaining walls are often used; in the construction of buildings having basements, roads, or bridges when it is necessary to retain embankments or earth in a relatively vertical position. Retaining walls are commonly supported by soil (or rock) underlying the base slab, or supported on piles; as in case of bridge abutments and where water may erode or undercut the base soil as in water front structures.

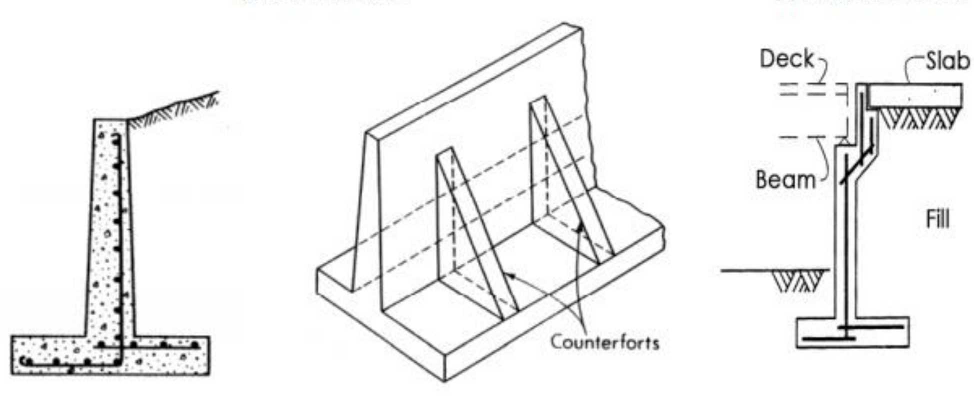
TYPES OF RETAINING WALLS

There are many types of retaining walls; they are mainly classified according to their behavior against the soil:-

a) Gravity retaining walls are constructed of plain concrete or stone masonry. They depend mostly on their own weight and any soil resting on the wall for stability. This type of construction is not economical for walls higher than 3m.
b) Semi-gravity retaining walls are modification of gravity wall in which small amounts of reinforcing steel are introduced for minimizing the wall section.
c) Cantilever retaining walls are the most common type of retaining walls and are generally used for wall high up to 8m. It derives its name from the fact that its individual parts behave as, and are designed as, cantilever beams. Its stability is a function of strength of its individual parts.
d) Counterfort retaining walls are similar to cantilever retaining walls, at regular intervals, however, they have thin vertical concrete slabs behind the wall known as counterforts that tie the wall and base slab together and reduce the shear and bending moment. They are economical when the wall height exceeds 8m. Whereas, if bracing is in front of the wall and is in compression instead of tension, the wall is called Buttress retaining wall.
e) Bridge abutments are special type of retaining walls, not only containing the approach fill, but serving as a support for the bridge superstructure.



(a) GRAVITY WALLS (b) SEMI-RAVITY WALL



(c) CANTILEVER WALL (d) COUNTERFORT WALL (e) BRIDGE ABUTMENT

Cantilever Wall:

The 'cantilever wall' is the most common type of retaining structure and is generally economical for heights up to about 8 m. The structure consists of a vertical stem, and a base slab, made up of two distinct regions, viz. a heel slab and a toe slab. All three components behave as one-way cantilever slabs: the 'stem' acts as a vertical cantilever under the lateral earth pressure; the 'heel slab' acts as a (horizontal) cantilever under the action of the weight of the retained earth (minus soil pressure acting upwards from below); and the 'toe slab' also acts as a cantilever under the action of the resulting soil pressure (acting upward). The detailing of reinforcement (on the flexural tension faces) is accordingly as depicted in Fig. below. The stability of the wall is maintained essentially by the weight of the earth on the heel slab plus the self weight of the structure.

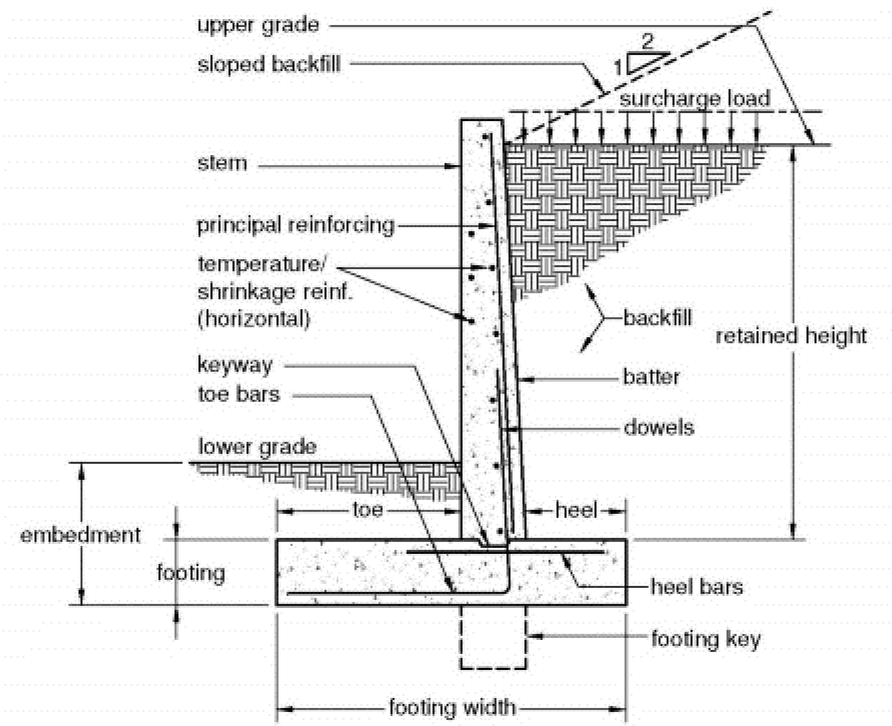


Fig: Cantilever Retaining Wall

Counterfort Wall:

For large heights, in a cantilever retaining wall, the bending moments developed in the stem, heel slab and toe slab become very large and require large thicknesses. The bending moments (and hence stem/slab thicknesses) can be considerably reduced by introducing transverse supports, called counterforts, spaced at regular intervals of about one-third to one-half of the wall height), interconnecting the stem‡ with the heel slab. The counterforts are concealed within the retained earth (on the rear side of the wall). Such a retaining wall structure is called the counterfort wall, and is economical for heights above (approx.) 7 m. The counterforts subdivide the vertical slab (stem) into rectangular panels and support them on two sides (suspender-style), and themselves behave essentially as vertical cantilever beams of T-section and varying depth. The stem and heel slab panels between the counterforts are now effectively 'fixed' on three sides (free at one edge), and for the stem the predominant direction of bending (and flexural reinforcement) is now horizontal (spanning between counterforts), rather than vertical (as in the cantilever wall).

EARTH PRESSURES AND STABILITY REQUIREMENTS

Lateral Earth Pressures

The lateral force due to earth pressure constitutes the main force acting on the retaining wall, tending to make it bend, slide and overturn. The determination of the magnitude and direction of the earth pressure is based on the principles of soil mechanics.

In general, the behaviour of lateral earth pressure is analogous to that of a fluid, with the magnitude of the pressure p increasing nearly linearly with increasing depth z for moderate depths below the surface:

$$\mathbf{p = C\gamma_e z}$$

where γ_e is the unit weight of the earth and C is a coefficient that depends on its physical properties, and also on whether the pressure is active or passive. ‘Active pressure’ (p_a) is that which the retained earth exerts on the wall as the earth moves in the same direction as the wall deflects. On the other hand, ‘passive pressure’ (p_p) is that which is developed as a resistance when the wall moves and presses against the earth (as on the toe side of the wall). The coefficient to be used in Eq. 14.9 is the active pressure coefficient, C_a, in the case of active pressure, and the passive pressure coefficient, C_p, in the case of passive pressure; the latter (C_p) is generally much higher than the former (C_a) for the same type of soil.

In the absence of more detailed information, the following expressions for Ca and C_p, based on Rankine’s theory, may be used for cohesionless soils and level backfills:

$$C_a = 1 - \sin\phi / 1 + \sin\phi$$

$$C_p = 1 + \sin\phi / 1 - \sin\phi$$

where φ is the angle of shearing resistance (or angle of repose). For a typical granular soil (such as sand), $\varphi \approx 30^o$, corresponding to which, $C_a = 1/3$ and $C_p = 3.0$. When the backfill is sloped†, the expression for C_a should be modified as follows:

$$C_a = \left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right]\cos\theta$$

where θ is the angle of inclination of the backfill, i.e., the angle of its surface with respect to the horizontal.

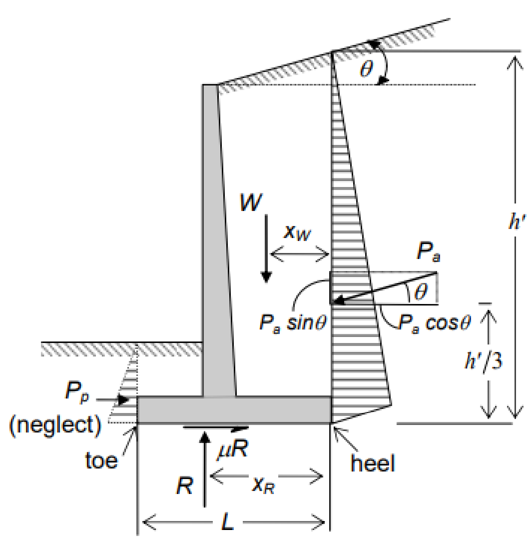


Fig: Forces acting on Cantilever Retaining wall

The direction of the active pressure, p_a, is parallel to the surface of the backfill. The pressure has a maximum value at the heel, and is equal to $C_a\gamma_e h'$, where h' is the height of the backfill, measured vertically above the heel.

For the case of a level backfill, $\theta = 0$ and h' = h, and the direction of the lateral pressure is horizontal and normal to the vertical stem. The force, P_a, exerted by the active earth pressure, due to a backfill of height h' above the heel, is accordingly obtained from the triangular pressure distribution as: $P_a = C_a\gamma_e (h')^2/2$

This force has units of kN per m length of the wall, and acts at a height h′/3 above the heel at an inclination θ with the horizontal. The force, P_p, developed by passive pressure on the toe side of the retaining wall is generally small (due to the small height of earth) and usually not included in the design calculations, as this is conservative.

Effect of Surcharge on a Level Backfill

Frequently, gravity loads act on a level backfill due to the construction of buildings and the movement of vehicles near the top of the retaining wall. These additional loads can be assumed to be static and uniformly distributed on top of the backfill, for calculation purposes. This distributed load w_s (kN/m^2) can be treated as statically equivalent to an additional (fictitious) height, $h_s = w_s/\gamma_e$, of soil backfill with unit weight γ_e. This additional height of backfill is called surcharge, and is expressed either in terms of height h_s, or in terms of the distributed load w_s

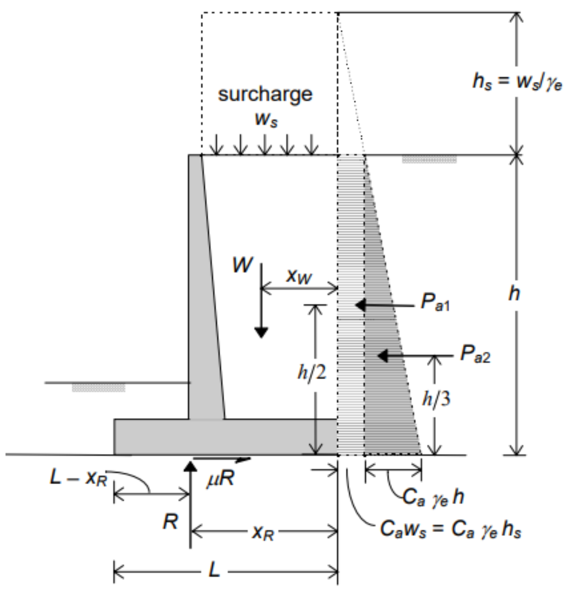


Fig: Effect of Surcharge on the level backfill

The presence of the surcharge not only adds to the gravity loading acting on the heel slab, but also increases the lateral pressure on the wall by $C_a\gamma_e h_s = C_a w_s$. The resulting trapezoidal earth pressure distribution is made up of a rectangular pressure distribution, superimposed on the triangular pressure distribution due to the actual backfill. The total force due to active pressure acting on the wall is accordingly given by:

$$P_a = P_{a1} + P_{a2}$$

$$P_{a1} = C_a w_s\, h = C_a\, \gamma_e\, h_s\, h$$

$$P_{a2} = C_a\, \gamma_e\, h^2/2$$

with the lines of action of P_{a1} and P_{a2} at h/2 and h/3 above the heel.

Stability Requirements

The Code (Cl. 20) specifies that the factors of safety against overturning (Cl. 20.1) and sliding (Cl. 20.2) should not be less than 1.4. Furthermore, as the stabilising forces are due to dead loads, the Code specifies that these stabilising forces should be factored by a value of 0.9 in calculating the factor of safety, FS. Accordingly,

$$FS = \frac{0.9 \times (\text{stabilising force or moment})}{\text{destabilising force or moment}} \geq 1.4$$

Overturning

If the retaining wall structure were to overturn, it would do so with the toe acting as the centre of rotation. In an overturning context, there is no upward reaction R acting over the base width L. The expressions for the overturning moment M_o and the stabilising (restoring) moment M_r depend on the lateral earth pressure and the geometry of the retaining wall. For the case of a sloping backfill:

$$M_o = (P_a \cos\theta)(h'/3) = \left[C_a \gamma_e (h')^3/6\right] \cos\theta$$

$$M_r = W(L - x_w) + (P_a \sin\theta)L$$

where W denotes the total weight of the reinforced concrete wall structure plus the retained earth resting on the footing (heel slab), and x_w is the distance of its line of action from the heel.

For the case of a level backfill with surcharge: $M_o = P_{a1}(h/2) + P_{a2}(h/3)$

The factor of safety required against overturning is obtained as: $(FS)_{overturning} = \frac{0.9 M_r}{M_o} \geq 1.4$

Sliding

The resistance against sliding is essentially provided by the friction between the base slab and the supporting soil, given by F = μR

where R = W is the resultant soil pressure acting on the footing base and μ is the coefficient of static friction between concrete and soil. [In a sloping backfill, R will also include the vertical component of earth pressure, $P_a \sin\theta$. The value of μ varies between about 0.35 (for silt) to about 0.60 (for rough rock).

The factor of safety against sliding is obtained as:

$$(FS)_{sliding} = \frac{0.9F}{P_a \cos\theta}, \text{ which should be} \geq 1.4$$

When active pressures are relatively high (as when surcharge is involved), it will be generally difficult to mobilise the required factor of safety against sliding, by considering frictional resistance below the footing alone. In such a situation, it is advantageous to use a shear key projecting below the footing base and extending throughout the length of the wall. When the concrete in the 'shear key' is placed in an unformed excavation (against undisturbed soil), it can be expected to develop considerable passive resistance.

PROPORTIONING AND DESIGN OF CANTILEVER AND COUNTERFORT WALLS

Prior to carrying out a detailed analysis and design of the retaining wall structure, it is necessary to assume preliminary dimensions of the various elements of the structure using certain approximations. Subsequently, these dimensions may be suitably revised, if so required by design considerations.

Position of Stem on Base Slab for Economical Design

An important consideration in the design of cantilever and counterfort walls is the position of the vertical stem on the base slab. It can be shown that an economical design of the retaining wall can be obtained by proportioning the base slab so as to align the vertical soil reaction R at the base with the front face of the wall (stem). For this derivation, let us consider the typical case of a level backfill. The location of the resultant soil reaction, R, is dependent on the magnitude and location of the resultant vertical load, W, which in turn depends on the dimension X (i.e., the length of heel slab, inclusive of the stem thickness). For convenience in the derivation, X may be expressed as a fraction, α_x, of the full width L of the base slab ($X = \alpha_x L$). Assuming an average unit weight γ_e for all material (earth plus concrete) behind the front face of the stem (rectangle abcd), and neglecting entirely the weight of concrete in the toe slab,

$$R = W = \gamma_e h_X = \gamma e\ h\ (\alpha_X L)$$

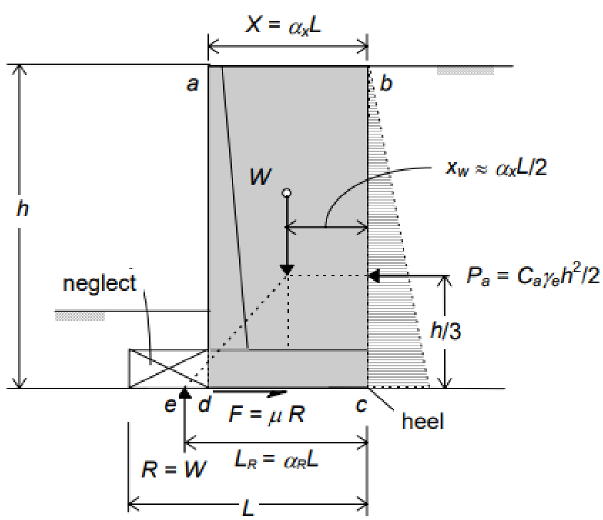


Fig: Proportioning of Retaining Wall

For a given location of R corresponding to a chosen value of X, the toe projection of the base slab (and hence its total width, L) can be so selected by the designer as to give any desired distribution of base soil pressure. Thus, representing the distance, LR, from the heel to R as a fraction α_R of base width L, the base pressure will be uniform if L is so selected as to make αR = 0.5. Similarly, for αR = 2/3, the base pressure distribution will be triangular. Thus, for any selected distribution of base pressure, αR is a constant and the required base width $L = L_R/\alpha_R$.

Considering static equilibrium and taking moments about reaction point e, and assuming $X_w \approx \alpha_X L/2$,

$$W(\alpha_R L - \alpha_X L/2) = P_a\, h/3$$

$$\Rightarrow \gamma_e h L^2 (\alpha_R \alpha_X - \alpha_X^2/2) = C_a \gamma_e h^3/6$$

$$\Rightarrow \frac{L}{h} = \sqrt{\frac{C_a/3}{2\alpha_R \alpha_X - \alpha_X^2}}$$

For economical proportioning for a given height of wall (h), the length of the base (L) must be minimum, i.e., L/h should be minimum. This implies that $(2\,\alpha_R\,\alpha_X - \alpha_X{}^2)$ should be maximum. The location of R, and hence the base width for any selected pressure distribution, is dependent on the variable X, i.e., αx. For maximising $(2\alpha_R\,\alpha_x - \alpha^2{}_x)$,

$$\alpha_X = \alpha_R$$

$$\Rightarrow \alpha_R L = \alpha_X L = X$$

Width of Base

Applying the above principle, an approximate expression for the minimum length of base slab for a given height of wall is obtained as:

$$\left(\frac{L}{h}\right)_{\min} \approx \frac{1}{\alpha_R}\sqrt{\frac{C_a}{3}}$$

$$\Rightarrow L_{\min} \approx (h/\alpha_R)\sqrt{C_a/3}$$

Alternatively, the minimum width of heel slab is given by:

$$X_{min} = \alpha_X L_{min} = \alpha_R L_{min} = h\sqrt{C_a/3}$$

The effect of surcharge or sloping backfill may be taken into account, approximately, by replacing h with h + h_s , or h'_s, respectively. Alternatively, and perhaps more conveniently, using the above principle, the heel slab width may be obtained by equating moments of W and P_a about the point d. The required L can then be worked out based on the base pressure distribution desired.

It may be noted that the total height h of the retaining wall is the difference in elevation between the top of the wall and the bottom of base slab. The latter is based on geotechnical considerations (availability of firm soil) and is usually not less than 1 m below the ground level on the toe side of the wall.

After fixing up the trial width of the heel slab (= X) for a given height of wall and backfill conditions, the dimension L may be fixed up. Initially, a triangular pressure distribution may be assumed, resulting in L = 3/2 X . Using other approximations related to stem thickness and base slab thickness, a proper analysis should be done to ascertain that

- the factor of safety against overturning is adequate;
- the allowable soil pressure, q_a, is not exceeded; and
- the factor of safety against sliding is adequate.

Condition (1) is generally satisfied; however, if it is not, the dimensions L and X may be suitably increased. If condition (2) is not satisfied, i.e., if qmax > q_a, the length L should be increased by suitably extending the length of the toe slab; the dimension X need not be changed. If condition (3) is not satisfied, which is usually the case, a suitable ‘shear key’ should be designed.

Proportioning and Design of Elements of Cantilever Walls

Initial Thickness of Base Slab and Stem

For preliminary calculations, the thickness of the base slab may be taken as about 8 percent of the height of the wall plus surcharge (if any); it should not be less than 300 mm. The base thickness of the vertical stem may be taken as slightly more than that of the base slab. For economy, the thickness may be tapered linearly to a minimum value (but not less than 150 mm) at the top of the wall; the front face of the stem is maintained vertical. If the length of the heel slab and/or toe slab is excessive, it will be economical to provide a tapered slab. With the above preliminary proportions, the stability check and determination of soil pressure (at the base) may be performed, and dimensions L and X of the base slab finalised. It may be noted that changes in thicknesses of base slab and stem, if required at the design stage, will be marginal and will not affect significantly either the stability analysis or the calculated (gross) soil pressures below the base slab.

Design of Stem, Toe Slab and Heel Slab

The three elements of the retaining wall, viz., stem, toe slab and heel slab have to be designed as cantilever slabs to resist the factored moments and shear forces. For this a load factor of 1.5 is to be used. In the case of the toe slab, the net pressure is obtained by deducting the weight of the concrete in the toe slab from the upward acting gross soil pressure. The net loading acts upward (as in the case of usual footings) and the flexural reinforcement has to be provided at the bottom of the toe slab. The critical section for moment is at the front face of the stem, while the critical section for shear is at a distance d from the face of the stem. A clear cover of 75 mm may be provided in base slabs. In the case of the heel slab, the pressures acting downward, due to the weight of the retained earth (plus surcharge, if any), as well as the concrete in the heel slab, exceed the gross soil pressures acting upward. Hence, the net loading acts downward, and the flexural reinforcement has to be provided at the top of the heel slab. The critical section for moment is at the rear face of the stem base.

In the case of the stem (vertical cantilever), the critical section for shear may be taken d from the face of the support (top of base slab), while the critical section for moment should be taken at the face of the support. For the main bars in the stem, a clear cover of 50 mm may be provided. Usually, shear is not a critical design consideration in the stem (unlike the base slab). The flexural reinforcement is provided near the rear face of the stem, and may be curtailed in stages for economy. Temperature and shrinkage reinforcement (A_{st},min = 0.12 percent of gross area) should be provided transverse to the main reinforcement. Nominal vertical and horizontal reinforcement should also be provided near the front face which is exposed.

Proportioning and Design of Elements of a Counterfort Wall

Initial Thicknesses of Various Elements

In a counterfort wall, counterforts are usually provided at a spacing of about one-third to one-half of the height of the wall. The triangular shaped counterforts are provided in the rear side of the wall, interconnecting the stem with the heel slab. Sometimes, small buttresses are provided in the front side below the ground level, interconnecting the toe slab with the lower portion of the stem. The presence of counterforts enables the use of stem and base slab thicknesses that are much smaller than those normally required for a cantilever wall. For preliminary calculations, the stem thickness and heel slab thickness may be taken as about 5 percent of the height of the wall, but not less than 300 mm. If the front buttress is provided, the thickness of the toe slab may also be taken as 0.05h; otherwise, it may be taken as in the case of the cantilever wall (0.08h). The thickness of the counterforts may be taken as about 6 percent of the height of the wall at the base, but not less than 300 mm. The thickness may be reduced along the height of the wall. With the above preliminary proportions, the stability check and determination of soil pressures (at the base) may be performed, and dimensions L and X of the base finalised, as in the case of the cantilever wall.

Each panel of the stem and heel slab, between two adjacent counterforts, may be designed as two-way slabs fixed on three sides, and free on the fourth side (free edge). These boundary conditions are also applicable to the toe slab, if buttresses are provided; otherwise the toe slab behaves as a horizontal cantilever, as in the case of the cantilever wall. The loads acting on these elements are identical to those acting on the cantilever wall discussed earlier. For the stem, bending in the horizontal direction between counterforts† is generally more predominant than bending in the vertical direction. Near the counterforts, the main reinforcement will be located close to the rear face of the stem, whereas midway between counterforts, the reinforcement will be close to the outside face. These two-ways slabs, subject to triangular/trapezoidal pressure distributions may be designed by the use of moment and shear coefficients (based on plate theory), available in various handbooks, and also in the IS Code for the design of liquid storage structures, viz., IS 3370 (Part 4). Alternatively, the slabs may be designed by the yield line theory. An alternative simplified method of analysis is demonstrated in example later on.

Design of Counterforts

The main counterforts should be firmly secured (by additional ties) to the heel slab, as well as to the vertical stem, as the loading applied on these two elements tend to separate them from the counterforts. In addition, the counterfort should be designed to resist the lateral (horizontal) force transmitted by the stem tributary to it. The counterfort is designed as a vertical cantilever, fixed at its base. As the stem acts integrally with the counterfort, the effective section resisting the cantilever moment is a flanged section, with the flange under compression. Hence, the counterforts may be designed as T-beams with the depth of section varying (linearly) from the top (free edge) to the bottom (fixed edge), and with the main reinforcement provided close to the sloping face. Since these bars are inclined (not parallel to the compression face), allowance has to be made for this in computing the area of steel required.

Example 1:

Determine suitable dimensions of a cantilever retaining wall, which is required to support a bank of earth 4.0 m high above the ground level on the toe side of the wall. Consider the backfill surface to be inclined at an angle of 15° with the horizontal. Assume good soil for foundation at a depth of 1.25 m below the ground level with a safe bearing capacity of 160 kN/m². Further assume the backfill to comprise granular soil with a unit weight of 16 kN/m³ and an angle of shearing resistance of 30°. Assume the coefficient of friction between soil and concrete to be 0.5.

SOLUTION

1. Data given: $h = 4.0 + 1.25 = 5.25$ m; $\mu = 0.5$

$\theta = 15^\circ$ $\quad \gamma_e = 16 \text{ kN/m}^3$

$\phi = 30^\circ$ $\quad q_a = 160 \text{ kN/m}^2$

- Earth pressure coefficients: $$C_a = \left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right]\cos\theta = 0.373$$

$$C_p = \frac{1+\sin\theta}{1-\sin\theta} = 3.0$$

2. Preliminary proportions:

- Thickness of footing base slab ≈ 0.08h = 0.08 × 5.25 = 0.42 m Assume a thickness of 420 mm.
- Assume a stem thickness of 450 mm at the base of the stem, tapering to a value of 150 mm at the top of the wall.
- For an economical proportioning of the length L of the base slab, it will be assumed that the vertical reaction R at the footing base is in line with the front face of the stem. For such a condition, (assuming the height above top of wall to be about 0.4 m), the length of the heel slab (inclusive of stem thickness):

$$X \approx \left(\sqrt{C_a/3}\right) h' = \sqrt{0.373/3}\,(5.25 + 0.4) \approx 2.0 \text{ m}$$

- Assuming a triangular base pressure distribution, L = 1.5X = 3.0 m
- The preliminary proportions are shown in Fig.

3. Stability against overturning

- Force due to active pressure: $P_a = C_a \gamma_e h'^2/2$

  where $h' = h + X\tan\theta$ [Fig. 14.28(a)]

  $= 5250 + 2000 \tan 15^\circ = 5786$ mm

  $P_a = (0.373)(16)(5.786)^2/2 = 99.9$ kN (per m length of wall)

  $\Rightarrow P_a \cos\theta = 99.9 \cos 15^\circ = 96.5$ kN

  $P_a \sin\theta = 99.9 \sin 15^\circ = 25.9$ kN

- Overturning moment $M_o = (P_a \cos\theta)h'/3 = (96.5)(5.786/3) = 186.1$ kNm
- Line of action of resultant of vertical forces [Fig. 14.28(a)] with respect to the heel can be located by applying statics, considering 1 m length of the wall:

force (kN)	distance from heel (m)	moment (kNm)
$W_1 = (16)(1.85)(5.25 - 0.42) = 143.0$	0.925	132.3
$W_2 = (16)(1.85)(0.5 \times 0.536) = 7.9$	0.617	4.9
$W_3 = (25)(0.15)(5.25 - 0.42) = 18.1$	1.925	34.8
$W_4 = (25 - 16)(4.83)(0.5 \times 0.30) = 6.5$	1.750	11.4
$W_5 = (25)(3.0)(0.42) = 31.5$	1.500	47.2
$P_a \sin\theta = 25.9$	0.000	0.0
$W = 232.9$		$M_W = 230.6$ kNm

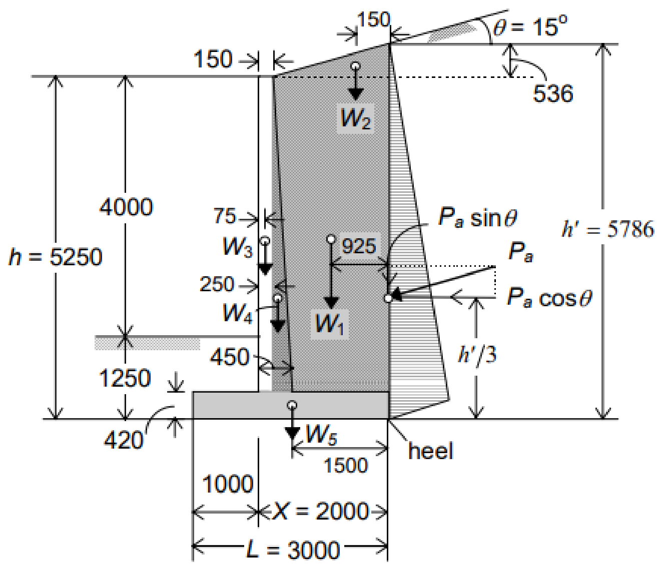
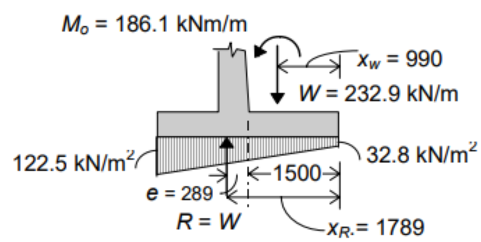
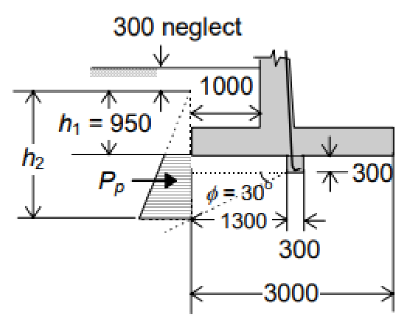


Fig: Forces on wall (with preliminary proportions)



(b) calculation of soil pressures



(c) design of shear key

⇒ distance of resultant vertical force from heel

$x_W = M_W / W = 230.6/232.9 = 0.990$ m

- Stabilising moment (about toe):

  $M_r = W (L - x_W)$

  $= 232.9 \times (3.0 - 0.99)$

  $= 468.1$ kNm (per m length of wall)

  $\Rightarrow (FS)_{overturning} = \dfrac{0.9 M_r}{M_o} = \dfrac{0.9 \times 468.1}{186.1} = 2.26 > 1.40$ — OK

4. **Soil Pressure at footing base:**

- resultant vertical reaction $R = W = 232.9$ kN (per m length of wall)
- distance of R from heel: $L_R = (M_W + M_o)/R$

  $= (230.6 + 186.1)/232.9 = 1.789$ m†
- eccentricity $e = L_R - L/2 = 1.789 - 3.0/2 = 0.289$ m, $< L/6 = 0.5$

- Hence, the resultant lies within the middle third of the base, which is desirable

$$\frac{6e}{L} = \frac{6 \times 0.289}{3.0} = 0.578$$

$$\Rightarrow q_{max} = \frac{R}{L}\left(1+\frac{6e}{L}\right) = \frac{232.9}{3.0}(1+0.578)$$

$$= 122.5 \text{ kN/m}^2 < q_a \qquad \text{— OK}$$

and $q_{min} = \frac{232.9}{3.0}(1 - 0.578) = 32.8 \text{ kN/m}^2$ [refer Fig. 14.28(b)]

5. Stability against sliding

- Sliding force = $P_a \cos\theta = 96.5$ kN
- Resisting force (ignoring passive pressure on the toe side) $F = \mu R$

$$= 0.5 \times 232.9 = 116.4 \text{ kN}$$

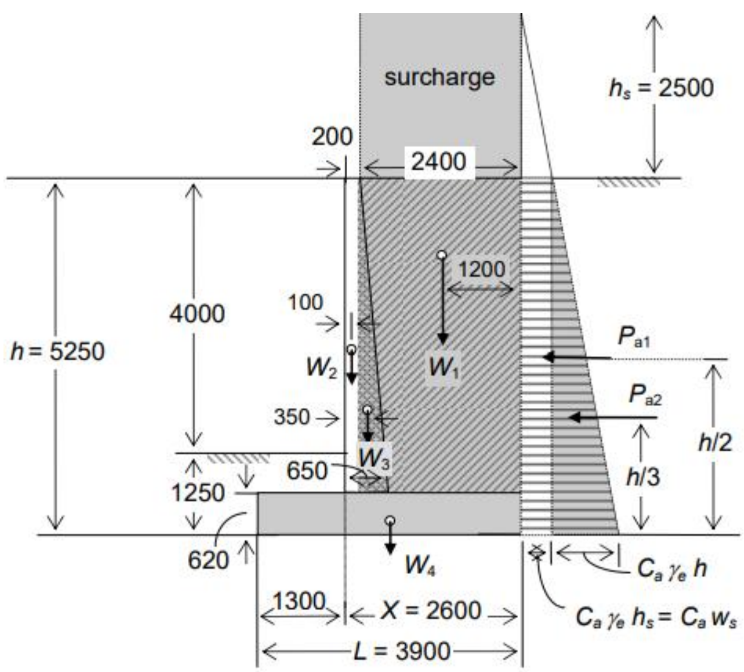
$$\Rightarrow (FS)_{sliding} = \frac{0.9F}{P_a \cos\theta} = \frac{0.9 \times 116.4}{96.5} = 1.085 < 1.40$$

- Hence, a *shear key* may be provided to mobilise the balance force through passive resistance.
- Assume a shear key 300 mm × 300 mm, at a distance of 1300 mm from toe as shown in Fig. 14.28(c). Distance $h_2 = 0.950 + 300 + 1.300 \tan 30^\circ = 2.001$ m

$P_p = C_p\gamma_e(h^2{}_2 - h^2{}_1)/2 = 3 \times 16 \times (2.001^2 - 0.95^2)/2$
$= 74.44$ kN

$$(F.S)_{sliding} = \frac{0.9(116.4 + 74.44)}{96.5} = 1.78 > 1.4 \qquad \text{— OK}$$

Example 2: Repeat the problem in Example 1, considering the backfill to be level, but subject to a surcharge pressure of 40 kN/m2 (due to the construction of a building). **Design the retaining wall** structure, assuming M 20 and Fe 415 steel.



(a) forces on wall (with preliminary proportions)

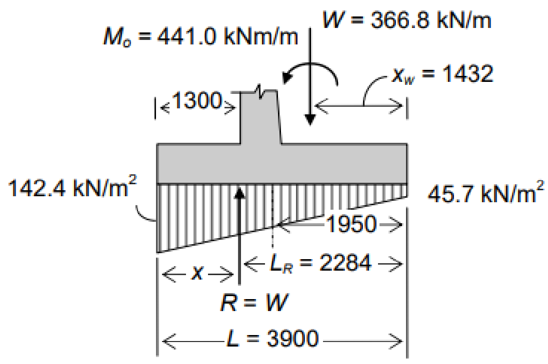


Fig. Calculation of Soil Pressures

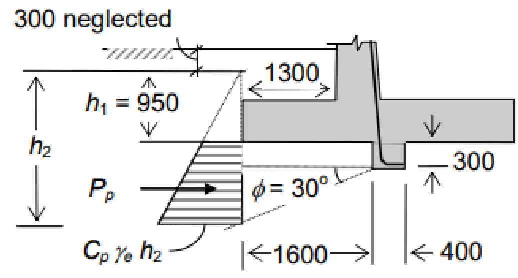


Fig. Design of Shear key

$h = 4.0 + 1.25 = 5.25$ m $\quad\quad \gamma_e = 16$ kN/m^3

$\phi = 30°$ $\quad\quad q_a = 160$ kN/m^2

$\mu = 0.5$ $\quad\quad w_s = 40$ kN/m^2

$\Rightarrow$ Equivalent height of earth as surcharge, $h_s = \dfrac{w_s}{\gamma_e} = \dfrac{40}{16} = 2.5$ m

$\Rightarrow h + h_s = 5.25 + 2.5 = 7.75$ m

- Earth pressure coefficients: $C_a = \dfrac{1-\sin\phi}{1+\sin\phi} = 1/3$

$$C_p = 1/C_a = 3.0$$

2. Preliminary proportions

- Thickness of footing base slab ≈ 0.08 (h + hs) = 0.08 × 7.75 = 0.620. Assume a thickness of 620 mm.
- Assume a stem thickness of 650 mm at the base of the stem, tapering to a value of 200 mm at the top of the wall.
- For an economical proportioning of the length L of the base slab, it will be assumed that the vertical reaction R at the footing base is in line with the front face of the stem. For such a condition, the length of the heel slab (inclusive of stem thickness).

$$X \approx \sqrt{C_a/3}\,(h + h_s) = \sqrt{(1/3)/3}\,(7.75) = 2.58 \text{ m}$$

- Assuming a triangular soil pressure distribution below the base, L = 1.5X = 1.5 × 2.6 = 3.9 m
- The preliminary proportions are shown in Fig.

3. Stability against overturning

- Forces due to active pressure (per m length of wall) [Fig. 14.29(a)]:
  $P_{a1} = C_a\, w_s\, h = (1/3)(40)(5.25) = 70.0$ kN
  $P_{a2} = C_a\, \gamma_e\, h^2/2 = (1/3)(16)(5.25)^2/2 = 73.5$ kN
  $\Rightarrow P_a = 70.0 + 73.5 = 143.5$ kN
- Overturning moment $M_o = P_{a1}\, h/2 + P_{a2}\, h/3$
  $\Rightarrow M_o = (70.0)(5.25/2) + (73.5)(5.25/3)$
  $= 312.4$ kNm (per m length of wall)
- Line of action of resultant of vertical forces [Fig. 14.29(a)] with respect to the heel can be located by applying statics, considering 1 m length of the wall:

force (kN)		distance from heel (m)	moment (kNm)
W_1 = (16)(2.40)(7.75 – 0.62)	= 273.8	1.20	328.6
W_2 = (25)(0.20)(4.63)	= 23.2	2.50	58.0
W_3 = (25 – 16) (0.5 × 0.45) (4.63)	= 9.4	2.25	21.1
W_4 = (25)(3.90)(0.62)	= 60.4	1.95	117.8
	W = 366.8		M_W = 525.5 kNm

⇒ distance of resultant vertical force from heel

$x_W = M_W/W = 525.5/366.8 = 1.432$ m

- Stabilising moment (about toe):

  $M_r = W(L - x_W)$
  $= 366.8 \times (3.9 - 1.432)$
  $= 905.3$ kNm (per m length of wall)

  $$\Rightarrow (FS)_{overturning} = \frac{0.9M_r}{M_o} = \frac{0.9 \times 905.3}{312.4} = 2.61 > 1.40 \quad \text{— OK}$$

4. Soil Pressure at footing Base:

- resultant vertical reaction $R = W = 366.8$ kN (per m length of wall)
- distance of R from heel: $L_R = (M_W + M_o)/R$
  $= (525.5 + 312.4)/366.8 = 2.284$ m†
- eccentricity $e = L_R - L/2 = 2.284 - 3.9/2 = 0.334$ m ($< L/6 = 0.65$)
  indicating that the resultant lies well inside the middle third of the base.

  $$\Rightarrow \frac{6e}{L} = \frac{6 \times 0.334}{3.9} = 0.514$$

  $$\Rightarrow q_{max} = \frac{R}{L}\left(1 + \frac{6e}{L}\right) = \frac{366.8}{3.9}(1 + 0.514)$$

  $= 142.4$ kN/m² $< q_a = 150$ kN/m² — OK.

  $$\Rightarrow q_{min} = \frac{R}{L}\left(1 - \frac{6e}{L}\right) = \frac{366.8}{3.9}(1 - 0.514) = 45.7 \text{ kN/m}^2,$$

5. Stability against sliding

- Sliding force = P_a = 143.5 kN (per m length of wall)
- Resisting force (ignoring passive pressure) $F = \mu R$
  $= 0.5 \times 366.8 = 183.4$ kN $> P_a$
- $$(F.S)_{sliding} = \frac{0.9F}{P_a} = \frac{0.9 \times 183.4}{143.5} = 1.15 < 1.4$$
- Hence, a *shear key* needs to be provided to generate the balance force through passive resistance.
  Required $P_p = 1.40 \times 143.5 - 0.9 \times 183.4 = 35.8$ kN (per m length of wall)
  Providing a shear key 300 mm × 400 mm at 1.6 m from toe [Fig. 14.29(c)],
  $h_2 = 0.95 + 0.3 + 1.6 \tan 30^\circ = 2.17$ m
  $P_p = 3 \times 16(2.17^2 - 0.95^2)/2 = 91.4$ kN

  $$\Rightarrow (F.S)_{sliding} = \frac{0.9(183.4 + 91.4)}{143.5} = 1.72 > 1.4 \quad \text{— OK}$$

6. Design of toe slab

- The loads considered for the design of the toe slab are as shown in fig. The net pressures, acting upward, are obtained by reducing the uniformly distributed self-weight of the toe slab from the gross pressures at the base. Self-weight loading = 25 × 0.62 = 15.5 kN/m^2
- The net upward pressure varies from 126.9 kN/m^2 to 94.7 kN/m^2
- Assuming a clear cover of 75 mm and 16 φ bars, d = 620 – 75 – 8 = 537 mm
- Applying a load factor of 1.5, the design shear force (at d = 537 mm from the front face of the stem) and the design moment at the face of the stem are given by:

$V_u \approx 1.5(126.9 + 94.7)/2 \times (1.3 - 0.537) = 126.8$ kN/m
$M_u = 1.5 \times [(94.7 \times 1.3^2/2) + (126.9 - 94.7) \times 0.5 \times 1.3^2 \times 2/3] = 147.2$ kNm/m

- Nominal shear stress $\tau_v = \frac{V_u}{bd} = \frac{126.8 \times 10^3}{10^3 \times 537} = 0.236$ MPa

For a τ_c = 0.24 MPa, the required pt = 0.10 with M 20 concrete

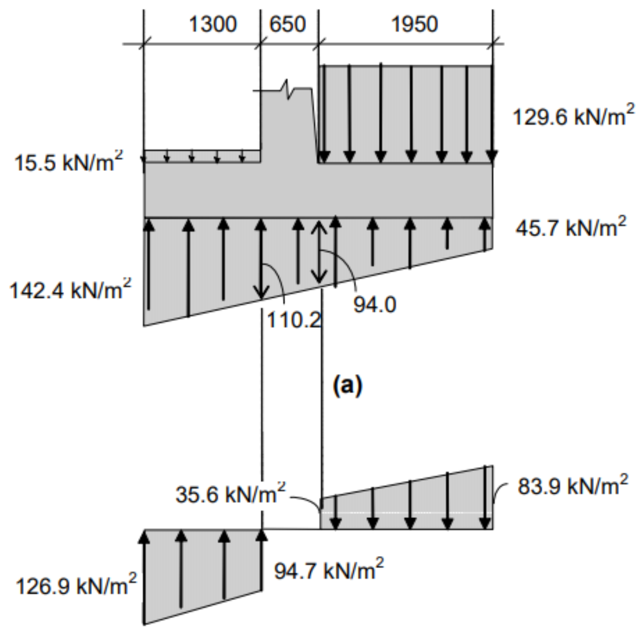


Fig: Net Soil pressures acting on base slab

- $R \equiv \frac{M_u}{bd^2} = \frac{147.2 \times 10^6}{10^3 \times 537^2} = 0.510$ MPa

$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - 4.598 \times 0.510/20}\right] = 0.15 \times 10^{-2}$, which is adequate for shear also

$\Rightarrow (A_{st})_{reqd} = (0.15 \times 10^{-2}) \times 10^3 \times 537 = 806$ mm^2/m

- Using 16 ϕ bars, spacing required = 201 × 10^3/806 = 249 mm
  Provide 16 ϕ bars @ 240 c/c at the bottom of the toe slab. The bars should extend by at least a distance L_d = 47.0 × 16 = 752 mm beyond the front face of the stem, on both sides. As the toe slab length is only 1.3 m overall, no curtailment of bars is resorted to here.

7. Design of Heel Slab

- The loads considered for the design of the heel slab are as shown in Fig. The distributed loading acting downward on the heel slab is given by
  - Overburden + surcharge @ 16 × (7.75 – 0.62) = 114.1 kN/m2
  - Heel slab @ 25 × 0.62 = 15.5 kN/m^2 ⇒ w = 129.6 kN/m^2
- The net pressure acts downwards, varying between 35.6 kN/m2 and 83.9 kN/m^2 as shown in Fig.
- Applying a load factor of 1.5, the design shear force and bending moment at the (rear) face of the stem are given by
  - Vu = 1.5(35.6 + 83.9)/2 × 1.95 = 174.8 kN/m
  - Mu = 1.5 × [(35.6 × 1.952 /2) + (83.9 – 35.6) × 0.5 × 1.952 × 2/3] = 193.4 kNm/m
- Assuming a clear cover of 75 mm and 16 φ bars, d = 620 – 75 – 8 = 537 mm
- Nominal shear stress $$\tau_v = \frac{V_u}{bd} = \frac{174.8 \times 10^3}{10^3 \times 537} = 0.326 \text{ MPa}$$

Corresponding τ_c = 0.33, with M 20 concrete; So $(p_t)_{reqd.}$= 0.20

- $$R \equiv \frac{M_u}{bd^2} = \frac{193.4 \times 10^6}{10^3 \times 537^2} = 0.670 \text{ MPa}$$
  $$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - 4.598 \times 0.670/20}\right] = 0.193 \times 10^{-2} < 0.20 \times 10^{-2} \text{ required for shear}$$
  $$\Rightarrow (A_{st})_{reqd} = (0.20 \times 10^{-2}) \times 10^3 \times 537 = 1074 \text{ mm}^2/\text{m}$$

- Using 16 φ bars, spacing required = 201 × 103 /1074 = 187 mm
  Provide 16 φ bars @ 180 c/c at the top of the heel slab. The bars should extend by at least a distance Ld = 47.0 × 16 = 752 mm beyond the rear face of the stem, on both sides. The bars may be curtailed part way to the heel; however, since the length is relatively short, this is not resorted to in this example.

8. Design of vertical stem

- Height of cantilever above base h = 5.250 – 0.62 = 4.63 m
- Assuming a clear cover of 50 mm and 20 φ bars, d (at the base) = 650 – 50 – 10 = 590 mm
- Assuming a load factor of 1.5, maximum design moment
  $$M_u = 1.5[C_a w_s h^2/2 + C_a \gamma_e h^3/6] = 1.5 \times (1/3)[40 \times 4.63^2/2 + 16 \times 4.63^3/6] = 346.7 \text{ kNm/m}$$
  $$\Rightarrow R \equiv \frac{M_u}{bd^2} = \frac{346.7 \times 10^6}{10^3 \times 590^2} = 1.00 \text{ MPa}$$
  $$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - 4.598 \times 1.00/20}\right] = 0.295 \times 10^{-2}$$
  $$\Rightarrow (A_{st})_{reqd} = (0.295 \times 10^{-2}) \times 10^3 \times 590 = 1741 \text{ mm}^2/\text{m}$$
- Using 16 ϕ bars, spacing required = $\frac{201 \times 10^3}{1741}$ = 115 mm
  Provide 16 ϕ @ 110 c/c, bars extending into the 'shear key'. [This anchorage will be more than the minimum required: L_d = 47.0 × 16 = 752 mm]

- **Check for Shear:**

Critical section is at $d = 0.59$ m above base, i.e., at $z_s = 4.63 - 0.59 = 4.04$ m below top edge. Shear force at critical section $= 1.5\ [C_a w_s z_s + C_a \gamma_s z_s^2/2]$

$$= 1.5 \times (1/3)[40 \times 4.04 + 16 \times 4.04^2/2]$$
$$= 146 \text{ kN/m}$$

$$\tau_v = \frac{146 \times 10^3}{10^3 \times 590} = 0.248 \text{ MPa} < \tau_c \text{ for } p_t = 0.295$$

— OK

Note that since the shear stress is low and flexural reinforcement ratio also is low, the thickness of stem at base could be reduced for a more economical design.

- Curtailment of bars:
  The curtailment of the bars may be done in two stages (at one-third and two-third heights of the stem above the base) as shown in Fig. below. It can be verified that the curtailment satisfies the Code requirements.
- Temperature and Shrinkage reinforcement Provide two-thirds of the (horizontal) bars near the front face (which is exposed to weather and the remaining one-third near the rear face. For the lowermost onethird height of the stem above base,
  $A_{st} = (0.0012 \times 103 \times 650) \times 2/3 = 520\ mm^2$ /m
- Using 8 φ bars, spacing required = $50.3 \times 10^3 /520 = 97$ mm ≈ 100 mm. Provide 8 φ @ 100 c/c near front face and 8 φ @ 200 c/c near rear face in the lowermost one-third height of the wall; 8 φ @ 200 c/c near front face and 8 φ @ 400 c/c in the middle one-third height; and 8 φ @ 300 c/c near front face and 8 φ @ 600 c/c near the rear face in the top one-third height of the wall.
- Also provide nominal bars 10 φ bars @ 300 c/c vertically near the front face.

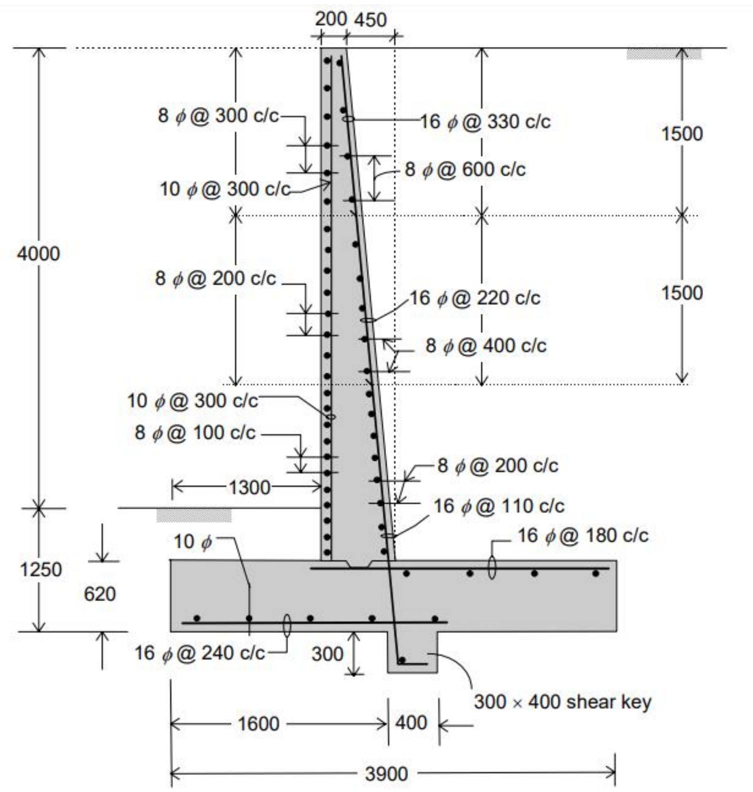


Fig: Detailing of cantilever wall

Counterfort Retaining Wall Numerical

Example 1:

Design a suitable counterfort retaining wall to support a level backfill, 7.5 m high above the ground level on the toe side. Assume good soil for foundation at a depth of 1.5 m below the ground level with a safe bearing capacity of 170 kN/m^2. Further assume the backfill to comprise granular soil with a unit weight of 16 kN/m^3 and an angle of shearing resistance of 30^o . Assume the coefficient of friction between soil and concrete to be 0.5. Use M 25 and Fe 415 steel.

Solution:

1. **Data given**: $h = 7.5 + 1.5 = 9.0$ m; $\mu = 0.5$
   $\theta = 0^o$ $\gamma_e = 16$ kN/m^3
   $\phi = 30^o$ $q_a = 170$ kN/m^2

- Earth pressure coefficients: $C_a = \dfrac{1-\sin\theta}{1+\sin\theta} = 0.333$

$$C_p = \frac{1+\sin\theta}{1-\sin\theta} = 3.0$$

2. Preliminary proportions:

- The (triangular shaped) counterforts are provided on the rear (backfill) side of the wall, interconnecting the stem with the heel slab.
  - Spacing of counterforts ≈ h 3 1 to h 2 1 = 3.0 m to 4.5 m
  - Assume the counterforts are placed with a clear spacing of 3.0 m.
  - Thickness of counterforts ≈ 0.05h = 0.05 × 9.0 = 0.45 m. Assume a thickness of 500 mm.
- Thickness of heel slab ≈ 0.05h = 0.05 × 9.0 = 0.45 m. Assume a thickness of 500 mm
- Assuming that the front buttresses are not provided, Thickness of toe slab ≈ 0.08h = 0.08 × 9.0 = 0.72 m. Assume a thickness of 720 mm
- Thickness of stem slab ≈ 0.06h = 0.06 × 9.0 = 0.54 m. Assume a stem thickness of 600 mm at the base of the stem, tapering to a value of 300 mm at the top of the wall.
- For an economical proportioning of the length L of the base slab, it is assumed that the vertical reaction R at the footing base is in line with the front face of the stem. For such a condition, (inclusive of stem thickness)

$$X \approx \left(\sqrt{C_a/3}\right) h = \sqrt{0.333/3}\ (9.0) = 3.0 \text{ m}$$

- Assuming a triangular base pressure distribution, L = 1.5X = 4.5 m
- The preliminary proportions are shown in Fig.

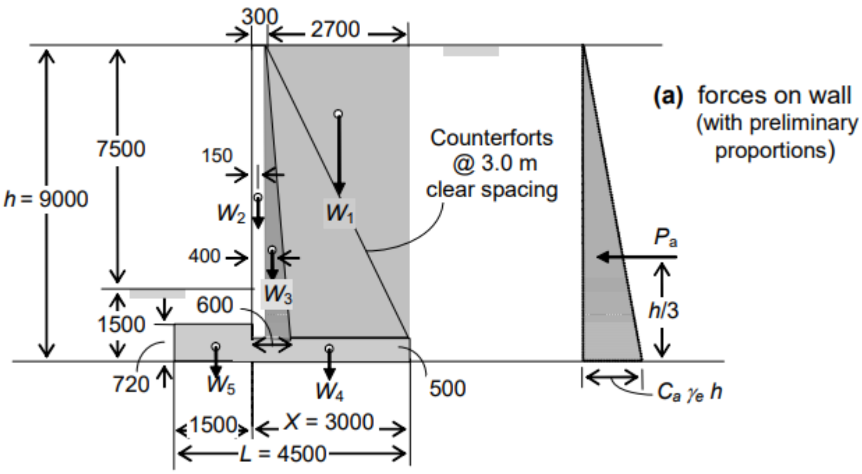
3. Stability against overturning

- Forces due to active pressure (per m length of wall)

  $P_a = C_a\, \gamma_e\, h^2/2 = (0.333)(16)(9.0)^2/2 = 216.0$ kN
- Overturning moment $M_o = P_a \times h/3$
  $\Rightarrow M_o = 216.0 \times (9.0/3)$
  $= 648.0$ kNm (per m length of wall)

- Line of action of resultant of vertical forces with respect to the heel can be located by applying statics, considering 1 m length of the wall (the marginal additional weight due to counterfort is ignored)

  ⇒ distance of resultant vertical force from heel $x_W = M_W / W = 864.8 / 506.9 = 1.706$ m



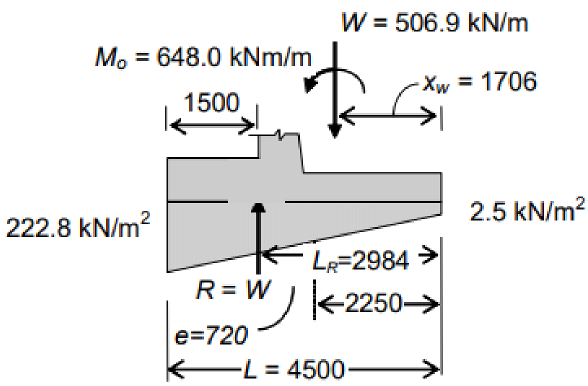


Fig: Cal. Of Soil Pressure

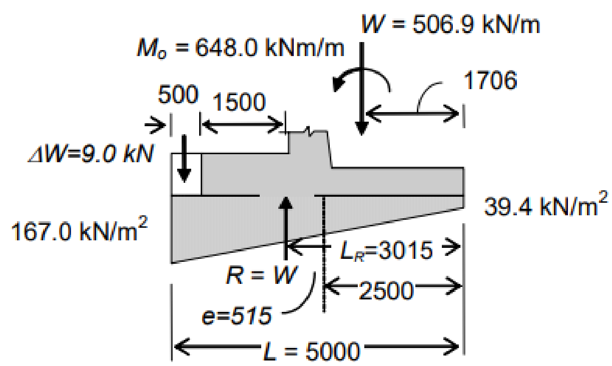


Fig: Revised design for safe soil pressures

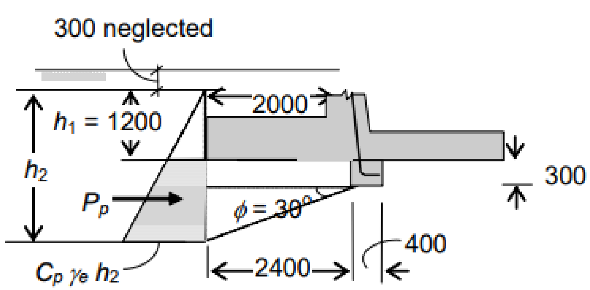


Fig: Design of Shear Key

force (kN)		distance from heel (m)	moment (kNm)
$W_1 = (16)(2.7)(9.0 - 0.5)$	= 367.20	1.35	495.72
$W_2 = (25)(0.3)(9.0 - 0.5)$	= 63.75	2.85	181.70
$W_3 = (25\text{-}16)(0.5)(0.3)(9.0 - 0.5)$	= 11.48	2.60	29.85
$W_4 = (25)(3.0)(0.5)$	= 37.50	1.50	56.30
$W_5 = (25)(1.5)(0.72)$	= 27.00	3.75	101.25
	W = 506.9		M_W = 864.8

- Stabilising moment (about toe):

  $M_r = W(L - x_W)$
  $= 506.9 \times (4.5 - 1.706)$
  $= 1416.4$ kNm (per m length of wall)

  $\Rightarrow (FS)_{overturning} = \dfrac{0.9M_r}{M_o} = \dfrac{0.9 \times 1416.4}{648.0} = 1.967 > 1.40$ — OK

4. Soil Pressure at Footing Base:

- resultant vertical reaction $R = W = 506.9$ kN (per m length of wall)
- distance of R from heel: $L_R = (M_W + M_o)/R$
  $= (864.8 + 648.0) / 506.9 = 2.984$ m†
- eccentricity $e = L_R - L/2 = 2.984 - 4.5/2 = 0.734$ m $(< L/6 = 0.75)$
  indicating that the resultant lies well inside the middle third of the base.

  $\Rightarrow \dfrac{6e}{L} = \dfrac{6 \times 0.734}{4.5} = 0.978$

  $\Rightarrow q_{min} = \dfrac{R}{L}\left(1 - \dfrac{6e}{L}\right) = \dfrac{506.9}{4.5}(1 - 0.978) = 2.5 \text{ kN/m}^2 > 0$ — OK

  $\Rightarrow q_{max} = \dfrac{R}{L}\left(1 + \dfrac{6e}{L}\right) = \dfrac{506.9}{4.5}(1 + 0.978)$

  $= 222.8 \text{ kN/m}^2 > q_a = 170 \text{ kN/m}^2$ — **UNSAFE.**

  Hence, the length of the base slab needs to be suitably increased on the toe side – say, by 500 mm.

  Let L = 5.0 m. Additional weight due to 500 mm extension of toe slab $\Delta W = 25 \times 0.5 \times 0.72 = 9.0$ kN

  $\Rightarrow R = W + \Delta W = 506.9 + 9.0 = 515.9$ kN

  Considering moments about the heel: $515.9\, L_R = 864.8 + (9.0)(5.0 - 0.25) + 648.0 \Rightarrow L_R = 3.015$ m

- Revised eccentricity $e = L_R - L/2 = 3.015 - 5.0/2$
  $= 0.515$ m $(< L/6 = 0.83)$

  $\Rightarrow \dfrac{6e}{L} = \dfrac{6 \times 0.515}{5.0} = 0.618$

  $\Rightarrow q_{min} = \dfrac{R}{L}\left(1 - \dfrac{6e}{L}\right) = \dfrac{515.9}{5.0}(1 - 0.618) = 39.4 \text{ kN/m}^2 > 0$ — OK

  $\Rightarrow q_{max} = \dfrac{R}{L}\left(1 + \dfrac{6e}{L}\right) = \dfrac{515.9}{5.0}(1 + 0.618) = 167.0 \text{ kN/m}^2$

  $< q_a = 170 \text{ kN/m}^2$ — OK.

5. Stability against sliding

- Sliding force $= P_a = 216.0$ kN (per m length of wall)
- Resisting force (ignoring passive pressure) $F = \mu R$
  $= 0.5 \times 515.9 = 257.9 \text{ kN} > P_a$
- $(F.S)_{sliding} = \dfrac{0.9F}{P_a} = \dfrac{0.9 \times 257.9}{216.0} = 1.075 < 1.4$ — **UNSAFE.**
- Hence, a *shear key* needs to be provided to generate the balance force through passive resistance.

  Required $P_p = 1.4 \times 216.0 - 0.9 \times 257.9 = 70.3$ kN (per m length of wall)

Providing a shear key 400 mm × 300 mm at 2.4 m from toe,

$h_2 = 1.2 + 0.3 + 2.4 \tan 30^o = 2.89$ m $\qquad$ $P_p = 3 \times 16(2.89^2 - 1.2^2)/2 = 165.9$ kN

$$\Rightarrow (F.S)_{sliding} = \frac{0.9\times(257.9+165.9)}{216.0} = 1.766 > 1.4 \qquad \text{— OK}$$

6. Design of toe slab

- The loads considered for the design of the toe slab are as shown in Fig.
- The net pressures, acting upward, are obtained by reducing the uniformly distributed self-weight of the toe slab from the gross pressures at the base. Self-weight loading = 25 × 0.72 = 18.0 kN/m^2

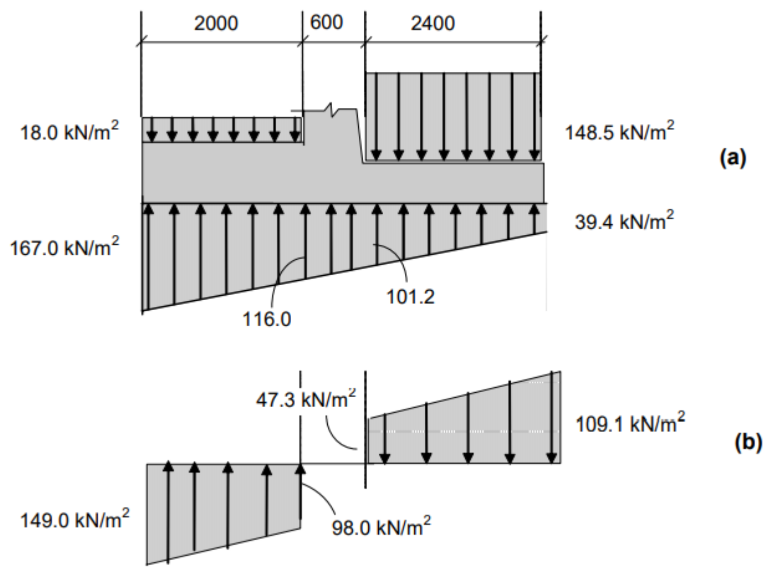


Fig: Net soil pressures acting on base slab

- The net upward pressure varies from 149.0 kN/m2 to 97.9 kN/m^2, as shown above.
- Assuming a clear cover of 75 mm and 16 φ bars, d = 720 – 75 – 8 = 637 mm
- Applying a load factor of 1.5, the design shear force (at d = 637 mm from the front face of the stem) and the design moment at the face of the stem are given by:

  $V_u \approx 1.5 \times (149.0 + 97.9)/2 \times (2.0 - 0.637) = 252.4$ kN/m
  $M_u = 1.5 \times [(97.9 \times 2.0^2/2) + (149.0 - 97.9) \times 0.5 \times 2.0^2 \times 2/3] = 395.9$ kNm/m

- Nominal shear stress $\tau_v = \frac{V_u}{bd} = \frac{252.4\times10^3}{10^3\times637} = 0.396$ MPa

  For a $\tau_c = 0.396$ MPa, the required $p_t = 0.32$ with M 25 concrete [refer Eq. 6.1]

- $R \equiv \frac{M_u}{bd^2} = \frac{395.9\times10^6}{10^3\times637^2} = 0.976$ MPa

  $$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2\times415}\left[1-\sqrt{1-4.598\times0.976/25}\right] = 0.284\times10^{-2}$$

  $< 0.32 \times 10^{-2}$ required for shear

$\Rightarrow (A_{st})_{reqd} = (0.32 \times 10^{-2}) \times 10^3 \times 637 = 2039$ mm²/m

- Using 16 φ bars, spacing required = 201 × 10^3 /2039 = 98.6 mm
- Using 20 φ bars, spacing required = 314 × 10^3 /2039 = 154 mm
- Provide 20 φ bars @ 150 c/c at the bottom of the toe slab.
- The bars should extend by at least a distance L_d = 47.0 × 20 = 940 mm beyond the front face of the stem, on both sides.
- Distribution steel: Provide 10 φ bars @ 200 c/c for the transverse reinforcement.

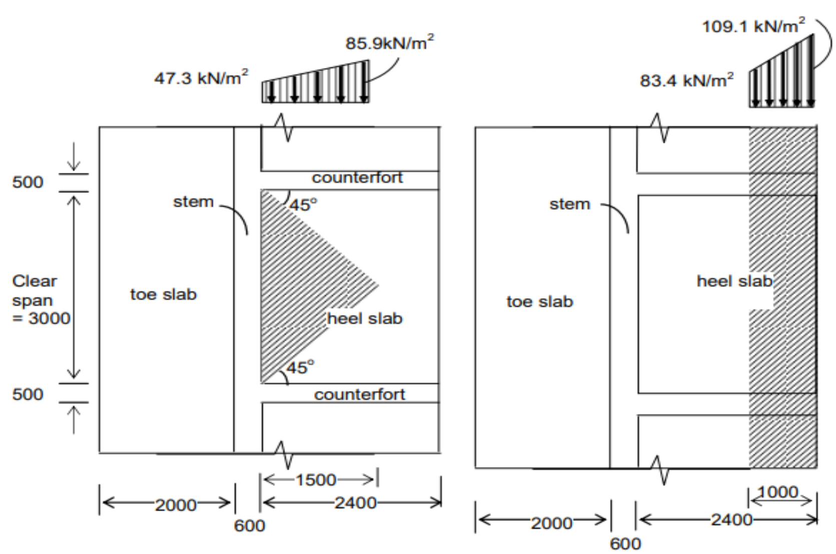
7. Design of heel slab

- The loads (net pressures) considered for the design of the heel slab are as shown in Fig. above.
- The distributed loading acting downward on the heel slab is given by
  - overburden @ 16 × (9.0 – 0.5) = 136.0 kN/m²
  - heel slab @ 25 × 0.5 = 12.5 kN/m² ⇒ w = 148.5 kN/m²
- The net pressure acts downwards, varying between 47.3 kN/m2 and 109.1 kN/m2 as shown in Fig.
- The counterforts are provided at a clear spacing of 3.0 m throughout the length of the wall. Thus, each heel slab panel (2.4m × 3.0m) may be considered to be fixed (continuous) at three edges (counterfort locations and junction with stem) and free at the fourth edge. The moment coefficients given in IS 456 do not cater to this set of boundary conditions, and reference needs to be made to other handbooks. Alternatively, we may apply the formulas obtained from yield line theory.
- A common simplified design practice is to assume that some tributary (triangular) portion of the net load acting on the heel slab is transmitted through cantilever action, while much of the load (particularly near the free edge) is transmitted in the perpendicular direction through continuous beam action. The reinforcements in the remaining regions are judiciously apportioned. This procedure is followed here.

• Design of heel slab for continuous beam action

Assuming a clear cover of 75 mm and 16 φ bars, d = 500 – 75 – 8 = 417 mm Consider a 1 m wide strip near the free edge of the heel. The intensity of pressure at a distance of 1 m from the free edge is 83.4 kN/m² .

Hence, the average loading on the strip may be taken as (83.4 + 109.1)/2 = 96.25 kN/m² . Applying a load factor of 1.5, w_u = 1.5 × 96.25 = 144.4 kN/m² . The effective span is given by l = 3.0 + 0.417 = 3.417 m.



(a) cantilever action **(b)** continuous beam action

Max. negative moment occurring in the heel slab at the counterfort location is given by

$M_{u,-ve} = w_u l^2/12 = 144.4 \times 3.417^2 /12 = 140.5$ kNm/m

Max. mid-span moment may be taken as

$M_{u,+ve} = w_u l^2/16 \approx 0.75 \times M_{u,-ve} = 105.4$ kNm/m

Design shear force

$V_u = w_u \times (\text{clear span} / 2 - d) = 144.4 \times (3.0/2 - 0.417) = 156.4$ kN/m

Design of top reinforcement (for –ve moments) at the counterforts

- Nominal shear stress $\tau_v = \dfrac{V_u}{bd} = \dfrac{156.4\times10^3}{10^3\times417} = 0.375$ MPa

  For a $\tau_c = 0.375$ MPa with M 25 concrete [refer Eq. 6.1], the required $p_t = 0.28$.

- $R \equiv \dfrac{M_u}{bd^2} = \dfrac{140.5\times10^6}{10^3\times417^2} = 0.808$ MPa

$$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2\times415}\left[1-\sqrt{1-4.598\times0.808/25}\right] = 0.233\times10^{-2} < p_t = 0.28$$

required for shear (in the absence of stirrups).

$\Rightarrow (A_{st})_{reqd} = (0.28 \times 10^{-2}) \times 10^3 \times 417 = 1168$ mm^2/m (required at 1m from the free edge)

- Using 16 φ bars, spacing required = 201 × 103 / 1168 = 172 mm
- Using 12 φ bars, spacing required = 113 × 103 / 1168 = 96 mm
- Minimum reinforcement for temperature and shrinkage:
  - Min. Ast = .012/100 * 1000 * 500 = 600 mm^2 /m < 1168 mm2 /m ----- OK.
- At a distance beyond 1m from the free edge, only minimum reinforcement need be provided:
  - Spacing of 12 φ bars required for min. reinf. = 113 × 103 / 600 = 188 mm
- Provide 12 φ bars @ 180 c/c at the top of the heel slab throughout, and introduce additional 12 φ bars in between two adjacent bars at the counterforts near the free edge over a distance of approx. 1m;
  - i.e., Provide 5 additional 12 φ bars on top, extending 1m from either side of the face of the counterfort.

Design of bottom reinforcement (for +ve moment) at mid-span of heel slab

- $R \approx 0.75 \times 0.808 = 0.606$ MPa

  $$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2\times415}\left[1-\sqrt{1-4.598\times0.606/25}\right] = 0.173\times10^{-2}$$

  $\Rightarrow (A_{st})_{reqd} = (0.173 \times 10^{-2}) \times 10^3 \times 417 = 721$ mm^2/m $> (A_{st})_{min} = 600$ mm^2/m

  Spacing of 12 ϕ bars required = 113 × 10^3 / 721 = 156 mm

  Provide **12 ϕ bars @ 150 c/c** at the bottom of the heel slab throughout.

  Distribution steel:

  Provide **10 ϕ bars @ 200 c/c** for the transverse reinforcement.

• Design of heel slab for cantilever action

Consider the triangular loading on the heel slab to be carried by cantilever action with fixity at the face of the stem.

- The intensity of load at the face of the stem = 47.3 kN/m^2.
- The intensity of load at a distance of 1.5m from the face of the stem is 85.9 kN/m^2.

- Total B.M. due to loading on the triangular portion $= \left(\frac{1}{2}\times 3.0\times 1.5\right)\times\left[47.3\times\frac{1.5}{3}+(85.9-47.3)\times\frac{1.5}{2\times 3}\right] = 74.93$ kNm

This moment is distributed non-uniformly across the width of 3.0m. For design purposes, the max. moment intensity (in the middle region) may be taken as two times the average value.

$\Rightarrow \quad M_{max} = 2 \times (74.93 / 3.0)$
$= 49.95$ kNm/m

$d = 417 - 12 = 405$mm

Applying a load factor of 1.5,

$$R \equiv \frac{M_u}{bd^2} = \frac{1.5\times 49.95\times 10^6}{1000\times 405^2} = 0.457 \text{ MPa}$$

$$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2\times 415}\left[1-\sqrt{1-4.598\times 0.457/25}\right] = 0.130\times 10^{-2}$$

$\Rightarrow (A_{st})_{reqd} = (0.130 \times 10^{-2}) \times 10^3 \times 405 = 527$ mm^2/m $< (A_{st})_{min} = 600$ mm^2/m (for temperature and shrinkage)

Provide **12 ϕ bars @ 180 c/c** at the top of the heel slab throughout.

8. Design of vertical stem

The simplified analysis procedure adopted for the heel slab is used here for the vertical stem also. The cantilever action is limited to the bottom region only (triangular portion) with fixity at the junction of the stem with the base slab. Elsewhere, the stem is treated as a continuous beam spanning between the counterforts. The bending moments reduce along the height of the stem, owing to the reduction in the lateral pressures with increasing height.

Height of stem above base h = 9.0 – 0.5 = 8.5 m.

Intensity of earth pressure at the base of the stem is

$p_a = C_a\, \gamma_e\, h = (0.333)(16)(8.5)$
$= 45.33$ kN/m^2 (linearly varying to zero at the top)

Applying a load factor of 1.5, $w_u = 1.5 \times 45.33 = 68.0$ kN/m^2 at base.

Clear spacing between the counterforts = 3.0 m.

- **Design of stem for continuous beam action**

At base

Assuming a clear cover of 50 mm and 20 ϕ bars,
$d = 600 - 50 - 10 = 540$ mm and effective span, $l = 3.0 + 0.54 = 3.54$ m

Max. –ve moment occurring in the stem at the counterfort location is given by
$M_{u,-ve} = w_u l^2/12 = 68.0 \times 3.54^2 /12 = 71.0$ kNm/m

Max. mid-span moment may be taken as
$M_{u,+ve} = w_u l^2/16 \approx 0.75 \times M_{u,-ve} = 53.3$ kNm/m

Design shear force
$V_u = w_u \times (\text{clearspan}/2 - d) = 68.0 \times (3.0/2 - 0.54) = 65.3$ kN/m

<u>Design of (rear face) reinforcement for –ve moments at the counterforts</u>

$$\Rightarrow R \equiv \frac{M_u}{bd^2} = \frac{71.0\times 10^6}{10^3\times 540^2} = 0.244 \text{ MPa}$$

$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2\times415}\left[1-\sqrt{1-4.598\times0.244/25}\right] = 0.068 \times 10^{-2}$

$\Rightarrow (A_{st})_{reqd} = (0.068 \times 10^{-2}) \times 10^3 \times 540 = 369$ mm²/m

Min. $A_{st} = \frac{0.12}{100}(1000)(600) = 720$ mm²/m > 369 mm²/m

Check for shear at base

$\tau_v = \frac{65.3\times10^3}{10^3\times540} = 0.121$ MPa < $\tau_c = 0.29$ MPa (for minimum $p_t = 0.15$) — OK

(Evidently, it is possible to reduce the thickness of the stem, for economy).

Design of (front face) reinforcement for +ve moments in the mid-span of stem

The minimum reinforcement requirement will govern the design on both faces, since $M_{u,+ve} < M_{u,-ve}$.

Using 12 ϕ bars, spacing required = 113 × 1000/ 720 = 156 mm

Provide **12 ϕ bars (horizontal) @ 150 c/c on both faces** of the stem (up to one-third height above base).

At one-third height above base

$d = 500 - 50 - 6 = 444$ mm and effective span $l = 3.444$m

$M_{u,-ve} = w_u l^2/12 = (68.0 \times 2/3) \times (3.444)^2 /12 = 44.81$ kNm/m

$\Rightarrow R \equiv \frac{M_u}{bd^2} = \frac{44.81\times10^6}{10^3\times444^2} = 0.227$ MPa

$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2\times415}\left[1-\sqrt{1-4.598\times0.227/25}\right] = 0.064 \times 10^{-2}$

$\Rightarrow (A_{st})_{reqd} = (0.064 \times 10^{-2}) \times 10^3 \times 444 = 282$ mm²/m

Min. $A_{st} = \frac{0.12}{100}(1000)(500) = 600$ mm²/m > 282 mm²/m

Using 12 ϕ bars, spacing required = 113 × 1000/ 600 = 188 mm

Provide **12 ϕ bars (horizontal) @ 180 c/c on both faces** of the stem (in the middle one-third height).

At two-thirds height above base

Min. $A_{st} = \frac{0.12}{100}(1000)(400) = 480$ mm²/m

Using 10 ϕ bars, spacing required = 78.5 × 1000/ 480 = 163 mm

Using 12 ϕ bars, spacing required = 113 × 1000/ 480 = 235 mm

Provide **12 ϕ bars (horizontal) @ 230 c/c on both faces** of the stem (in the upper one-third height).

9. Design of interior counterfort

The typical interior counterfort acts as a T beam of varying section cantilevering out of the base slab. The design should include:

- provision for beam action
- provision of horizontal ties against separation from stem
- provision of vertical ties against separation of base

Design of counterfort for T-beam action

The thickness of counterforts = 500 mm
Clear spacing of counterforts = 3.0 m
Thus, each counterfort receives earth pressure from a width of l= 3.0 + 0.5 = 3.5 m

At base

The intensity of earth pressure at the base of the stem is

$p_a = C_a \gamma_e h = (0.333)(16)(8.5) = 45.33 \text{ kN/m}^2$

Applying a load factor of 1.5,

$$M_u = 1.5 \times \left(\frac{1}{2} \times 45.33 \times 8.5\right) \times 3.5 \times \frac{8.5}{3} = 2866 \text{ kNm}$$

$$V_u = 1.5 \times \left(\frac{1}{2} \times 45.33 \times 8.5\right) \times 3.5 = 1012 \text{ kN}$$

From Fig. 14.37,
$\tan\theta = 2700/8500 \Rightarrow \theta = 17.6^{o}$
and $D_{base} = 2400 \times \cos\theta = 2287$ mm
Assuming a clear cover of 50 mm and 25 ϕ bars,
$d = 2287 - 50 - 12.5 = 2224$ mm

- *Effective flange width* (Cl 23.1.2 Code):

  $b_f = l_0/6 + b_w + 6D_f$ [Eq. 4.30]

  $= 8500/6 + 500 + (6 \times 600) = 5517$ mm,

  $b_f = b_w$ + clear span of slab

  $= 500 + 3000 = 3500$ mm

  Thus, $b_f = 3500$ mm (least of the above two values)

  Approximate requirement of tension steel is given by assuming a lever arm z to be the larger of $0.9d = 2001$mm and $d - D_f/2 = 1924$mm, i.e., 2001mm:

  $$(A_{st})_{reqd} = \frac{M_u}{0.87 f_y z} = \frac{2866 \times 10^6}{0.87(415)(2001)} = 3967 \text{ mm}^2$$

  No. of 25 ϕ bars required = $\frac{3967}{491} \approx 8$ bars (provide in two layers, with 25 ϕ spacer bars)

  $\Rightarrow d = 2287 - 50 - 25 - 12.5 = 2199$ mm

  Assuming the neutral axis to be located at $x_u = D_f$,

  $M_{uR} = 0.362 \times 25 \times 3500 \times 600 \times (2199 - 0.416 \times 600) = 37048 \times 10^6$ Nmm

  $> M_u = 2866 \times 10^6$ Nmm

  This clearly indicates that the neutral axis lies within the flange.

  $$R \equiv \frac{M_u}{bd^2} = \frac{2866 \times 10^6}{3500 \times 2199^2} = 0.169 \text{ MPa}$$

  $$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2 \times 415}\left[1 - \sqrt{1 - 4.598 \times 0.169/25}\right] = 0.047 \times 10^{-2}$$

  $\Rightarrow (A_{st})_{reqd} = (0.047 \times 10^{-2}) \times 3500 \times 2199 = 3639$ mm^2/m (which is close to the approximate value of 3967 mm^2 calculated)

Minimum reinforcement in a beam is given by $\frac{A_s}{bd} = \frac{0.85}{f_y}$

$\Rightarrow A_s = 0.85 \times 500 \times 2199 / 415 = 2252 \text{ mm}^2 < 3639 \text{ mm}^2$

Provide 8 nos 25 ϕ bars in two layers, four bars in each layer with a 25 mm separation.

Above one-third height from the base

The intensity of earth pressure at h (= 8.5 × 2 / 3) = 5.67m from top is

$p_a = C_a \gamma_e h = 45.33 \times 2/3 = 30.22 \text{ kN/m}^2$

Applying a load factor of 1.5,

$$M_u = 1.5 \times \left(\frac{1}{2} \times 30.22 \times 5.67\right) \times 3.5 \times \frac{5.67}{3} = 850 \text{ kNm}$$

$$V_u = 1.5 \times \left(\frac{1}{2} \times 30.22 \times 5.67\right) \times 3.5 = 450 \text{ kN}$$

$D_{h=5.67} = 2287 \times 2/3 = 1525$ mm

Assuming a clear cover of 50 mm and 25 ϕ bars,

$d = 1525 - 50 - 12.5 = 1462$ mm

Approximate requirement of tension steel is given by assuming a lever arm z to be the larger of $0.9d = 1316$mm and $d - D_f/2 = 1212$ mm, i.e., 1316 mm:

$$(A_{st})_{reqd} = \frac{M_u}{0.87 f_y z} = \frac{850 \times 10^6}{0.87(415)(1316)} = 1789 \text{ mm}^2$$

No. of 25 ϕ bars required $= \frac{1789}{491} \approx 4$ bars

$\Rightarrow d = 1462$ mm

Assuming the neutral axis to be located at $x_u = D_f$,

$$M_{uR} = 0.362 \times 25 \times 3500 \times 500 \times (1462 - 0.416 \times 500) = 19860 \times 10^6 \text{ Nmm} > M_u = 850 \times 10^6 \text{ Nmm}$$

This clearly indicates that the neutral axis lies within the flange.

$$R \equiv \frac{M_u}{bd^2} = \frac{850 \times 10^6}{3500 \times 1462^2} = 0.114 \text{ MPa}$$

$$\Rightarrow \frac{(p_t)_{reqd}}{100} = \frac{25}{2 \times 415}\left[1 - \sqrt{1 - 4.598 \times 0.114/25}\right] = 0.032 \times 10^{-2}$$

$\Rightarrow (A_{st})_{reqd} = (0.032 \times 10^{-2}) \times 3500 \times 1462 = 1638 \text{ mm}^2/\text{m}$ (which is close to the approximate value of 1789 mm² calculated)

Minimum reinforcement in a beam is given by $\frac{A_s}{bd} = \frac{0.85}{f_y}$

$\Rightarrow A_s = 0.85 \times 500 \times 1462 / 415 = 1497 \text{ mm}^2 < 1638 \text{ mm}^2$

Curtail 4 nos 25 φ bars and extend 4 nos 25 φ bars (rear face).

In order to satisfy the minimum reinforcement criteria, 4 nos 25 φ bars may be extended to the top of the counterfort, without any further curtailment.

Detailed Figure on next Page:

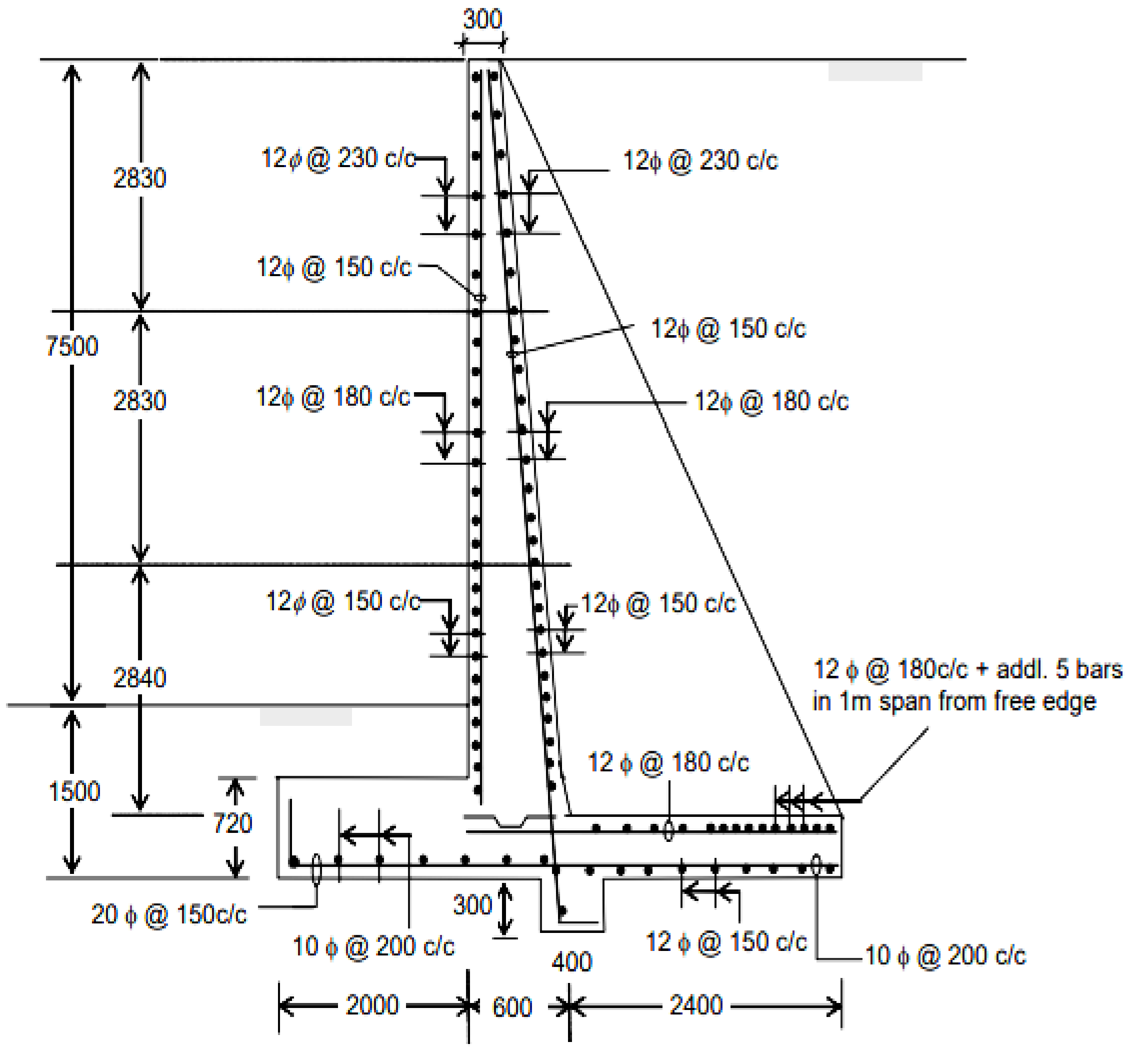


Fig: Reinforcement details of stem, toe slab and heel slab

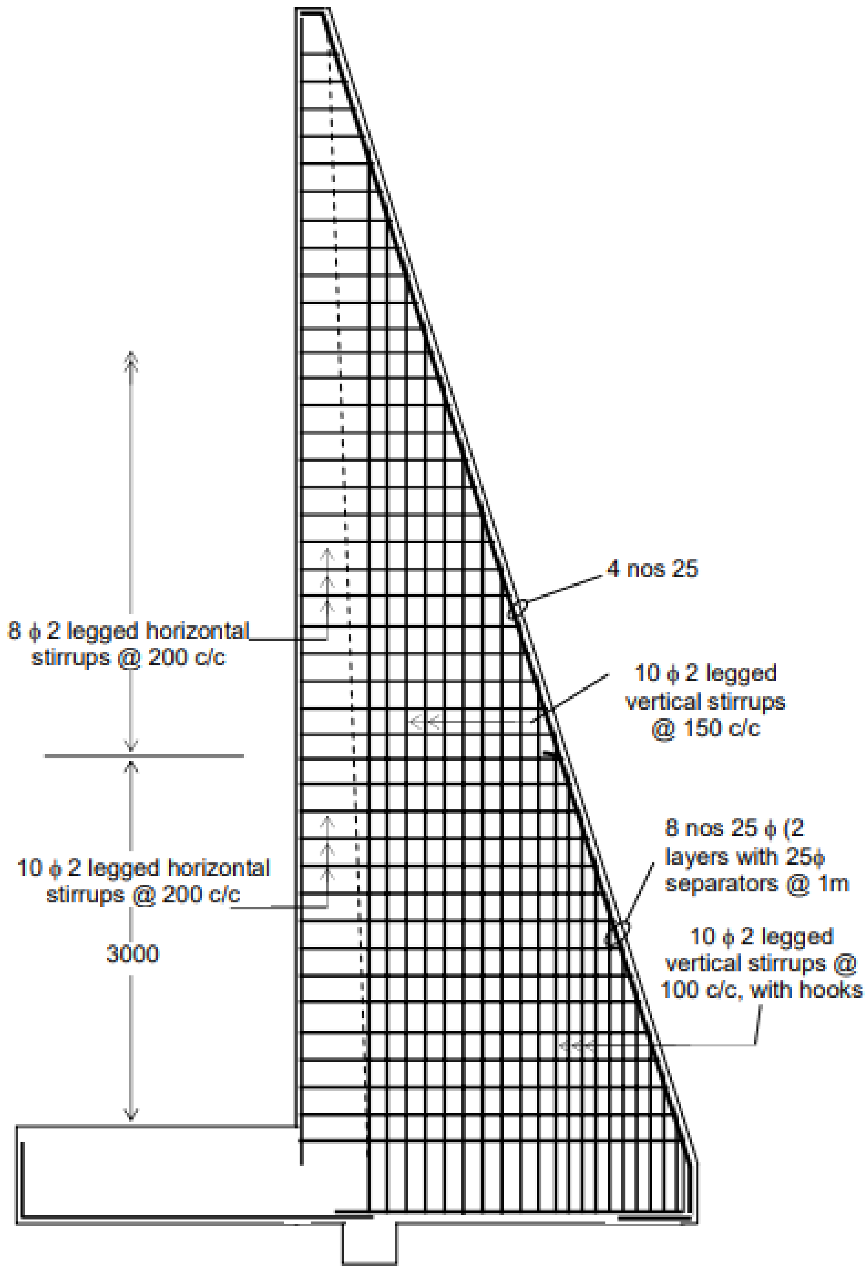


Fig: Section through counterfort showing counterfort reinforcement

WATER TANKS

Introduction

Tank (or reservoir) is a liquid storage structure that can be below or above the ground level. The liquid to be stored may be water, liquid petroleum, petroleum products or similar liquids. Though the tanks can be made of Reinforced Concrete (RC), steel or synthetic materials but in this unit only RC tanks have been covered. The tanks are often classified based on the following features:

- Shape: Circular (flexible or rigid base), Rectangular, Intze, Conical or Funnel etc.
- Location with respect to ground: Underground, Resting on ground, Partially underground and Overhead.
- Capacity: Large, Medium and Small.

The shape has a very important role to play because the structural behaviour of different components of the tank depends upon it. Flexure being predominant in rectangular tanks, its section are heavier in comparison to other shapes. But based on the economic considerations due to its simple form, rectangular tanks are sometimes prefered especially for small capacity.

Reservoir below ground level are normally built to store large quantities of water, whereas, the overhead type are built for direct distribution by gravity flow and are usually of smaller capacity. The capacity requirement of a tank helps in deciding what type of tank will be suitable.

General Design Consideration

Besides strength, water tightness is one of the main considerations in the design of RC water tanks. It has to be ensured in their design that the concrete does not crack on the water face. Minimum grade of concrete used in water tanks is M 20. Imperviousness of concrete can be ensured by implementing the following recommendations:

- Concrete mix containing well graded aggregate with water cement ratio less than 0.5 be used.
- Concrete should be richer in cement and very well compacted.
- Defects such as segregation and honey combing which are the potential source of leakage be avoided.

The crack of concrete can be controlled by adopting the following measures:

- The cracking due to shrinkage and temperature variation can be minirnised by keeping the concrete moist and filling the tank as soon as possible.
- Avoid the use of thick timber shuttering that prevents the easy escape of the heat of hydration from the concrete mass.
- Cracking is controlled by increasing the requirement of minimum reinforcement as given in Table below.

Table 22.1 : Minimum Reinforcement

Sl. No.	Nature	Percentage	+
		Mild Steel	HYSD
1.	Dummy concrete with no tension	0.15	0.12
2.	Concrete member with Thickness ≤ 100 mm Thickness ≥ 450 mm 100 mm≤ Thickness (= t mm, say) ≤ 450 mm	 0.30 0.20 $0.3-\left(\frac{t-100}{350}\right)\times 0.1$ (Linear interpolation)	 0.24 0.16 $0.24-\left(\frac{t-100}{350}\right)\times 0.08$ (Linear interpolation)

- Use of deformed bars or ribbed steel improves the level of cracking strains in concrete by even distribution and slip minimization.
- The cracking of concrete is also kept within allowable limits by reducing the allowable stresses in steel as given in Table below:

Table 22.2 : Allowable Stresses in Reinforcement

Sl.No.	Nature of Stress	Allowable Stress (MPa)	
		Plain Mild Steel*	HYSD
1.	Tension in steel placed within 225 mm from water face	100	150
2.	Tension in steel placed beyond 225 mm from water face	120	190
3.	Compression	120	190

*For deformed bars, increase by 20%.

To keep the concrete free from cracks, the tensile stress in concrete due to direct tension be limited to 0.27 $\sqrt{f_{ck}}$ MPa and due to bending tension to 0.37 $\sqrt{f_{ck}}$ MPa, where f_{ck} is the characteristic compressive strength of 150 mm cube of concrete in MPa. Design criteria for bending and direct tension are given below.

Design Criteria for Bending

Effective depth required, $d = \sqrt{\dfrac{M}{k\,b}}$

Area of flexural steel required, $A_{st} = \dfrac{M}{\sigma_{st}\, j\, d}$ (to be provided at flexural

Area of steel required, $A_s = \dfrac{T}{\sigma_{st}}$ (to be equally distributed in the cross-section)

Thickness of member can be calculated from :

$$\frac{T}{A_c + (m-1)\,A_s} \le \sigma_{ct}$$

σ_{ct} = Permissible direct tensile stress for concrete,

T = Direct tension,

σ_{cbc} = Permissible bending compressive stress for concrete,

σ_{st} = Permissible bending tensile stress for steel,

m = Modular ratio

$= \dfrac{280}{3\,\sigma_{cbc}}$,

j = Lever arm coefficient

$= 1 - \dfrac{n}{3}$,

where, M = Bending Moment (BM)

$k = \dfrac{1}{2}\,\sigma_{cbc}\, n\, j$

n = Neutral axis (N. A.) depth coefficient

$= \dfrac{1}{1 + \dfrac{\sigma_{st}}{m\,\sigma_{cbc}}}$

A_c = Area of concrete

$= b\,t$,

b = Width of the section, and

t = Thickness of the section.

- The concrete cover to the reinforcement is kept more for controlling the cracking. The minimum recommended cover is :

  Clear cover = 20 rnm or diameter of bar for steel in direct tension
  = 25 mrn for steel in bending tension
  = 30 mm for alternate drying and wetting condition

Joints in Water Tanks

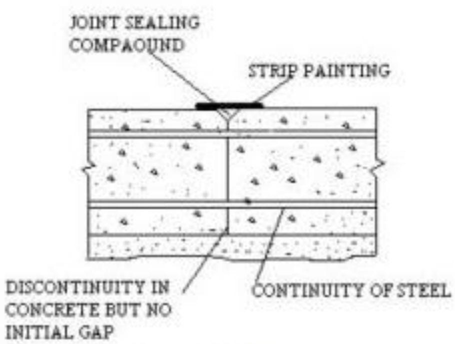
The various types of joints that are provided in water tanks can be categorized under three heads :

1. Movement Joints
2. Construction Joints
3. Temporary Open Joints

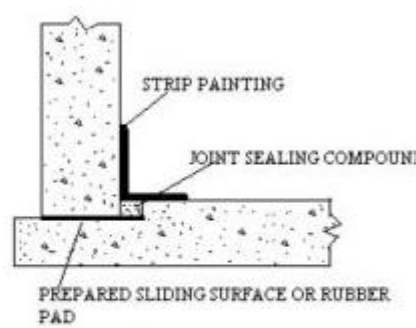
Movements Joints

These are provlded for accommodating relative movement of the two sides. All movement joints are essentially flexible joints and require the incorporation of special materials in order to maintain water tightness. The movement joints are of three types :

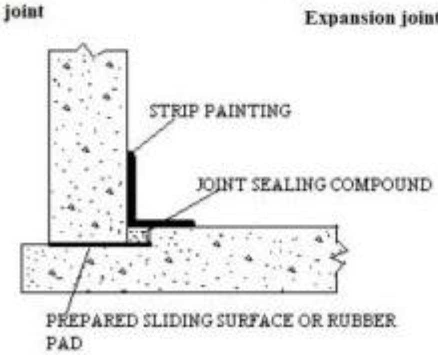
- Contraction Joint
- Expansion Joint
- Sliding Joint



Constraction joint



Expansion joint



Sliding joint

Design of tanks resting on ground

The water tanks resting on ground may be of the following types :

i. Circular Tank with Flexible Base
ii. Circular Tank with Rigid Base
iii. Rectangular Tank

If the floor slab is resting continuously on the ground, a minimum thickness of 150 mm may be provided with a nominal reinforcement of 0.24% HYSD steel bars (0.3% for MS bars) in each direction. The slab should rest on a 75 mm thick layer of lean concrete (M 10 mix). The layer of lean concrete.should be first cured and then it should be covered with a layer of tarfelt to enable the floor slab to act independent of the bottom layer of concrete.

The design of tank walls of various types has been discussed in the following sections :

i. Circular Tank with Flexible Base

Due to water pressure, the wall of a circular tank with flexible base between wall and base slab expand circurnferentially which increases linearly from zero at top to a maximum at the base as shown in Figure 22.4 by dotted line AB '. The circumferential expansion causes only hoop tension in the wall which will also increase linearly from top to the base of the wall. If D is the internal diameter of the tank, hoop tension at

depth h will be $\gamma h \frac{D}{2}$, here γ is the unit weight of water.

This will be zero at the top of the wall where h is zero and maximum at the base of the wall.

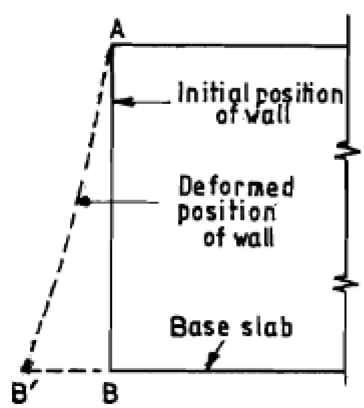


Fig. Circular Tank with Flexible base

ii. Circular Tank with Rigid Base

When the joint between the wall and face is rigid, there will not be any circumferential movement of the wall at its base and the wall will take the shape ACB (Figure below). The upper portion of the wall will have hoop tension while the lower part will bend like a cantilever fixed at B. In this case hoop tension will not be maximum at the base as in circular tank with flexible base. There are various methods for the determination of cantilever BM and hoop tension along the height of the wall. These methods are :

- Reissner's Method
- Carpenter's Simplified Method
- BIS Code Method
- Approximate Method

(a) *Reissner's Method*

According to this method the behaviour of circular tank with rigid base depends upon a parameter k defined by

$$k = \frac{48 H^4}{D^2 t^2}$$

where, H = Height of wall,

D = Inside diameter of tank, and

t = Thickness of wall of the tank.

Reissner developed tables from which cantilever BM at the base of the wall (Table 22.4), maximum hoop tension and its location (Table 22.5) are given in terms of the parameter k. The values are given for rectangular as well as triangular wall. Thus for a tepered wall of trapezoidal section a combination of the two may be considered. The maximum positive cantilever BM may be taken approximately 30% of the restraint BM i.e., cantilever BM at the base.

Table 22.4 : Reissner's Value of Restraint Moment

K	Rectangular Wall Section		Triangular Wall Section	
	Max. Tension	Height from Base	Max. Tension	Height from Base
0	0	—	—	—
10	$0.13\,P\left(\frac{D}{2}\right)$	10 H	$0.09\,P\left(\frac{D}{2}\right)$	0.65 H
100	$0.27\,P\left(\frac{D}{2}\right)$	1.0 H	$0.31\,P\left(\frac{D}{2}\right)$	0.58 H
1000	$0.47\,P\left(\frac{D}{2}\right)$	0.47 H	$0.52\,P\left(\frac{D}{2}\right)$	0.44 H
10000	$0.67\,P\left(\frac{D}{2}\right)$	0.31 H	$0.70\,P\left(\frac{D}{2}\right)$	0.30 H
∞	$1.0\,P\left(\frac{D}{2}\right)$	0	$1.0\,P\left(\frac{D}{2}\right)$	0

Table 22.5 : Reissner's Value of Hoop Tension

K	Rectangular Wall Section	Triangular Wall Section
0	$0.167\,pH^2$	$0.167\,pH^2$
10	$0.110\,pH^2$	$0.140\,pH^2$
100	$0.0582\,pH^2$	$0.0707\,pH^2$
1000	$0.024\,pH^2$	$0.026\,pH^2$
10000	$0.0085\,pH^2$	$0.009\,pH^2$
∞	0	0

(b) *Carpenter's Simplified Method*

Carpenter simplified the Reissner's method and gave the following expressions for the calculation of maximum cantilever BM, maximum hoop tension and its position :

Maximum cantilever BM, $M = F \gamma H^3$

Position of maximum Hoop Tension = $k H$ above base

The values of the coefficients F and K depend upon H/D and H/t ratios, and may be taken from Table 22.6.

Table 22.6 : Carpenter's Value of Coefficients F and K

Factor		F				K			
$\frac{H}{t} \rightarrow$		10	20	30	40	10	20	30	40
Values of H/D	0.2	0.046	0.028	0.022	0.015	—	0.50	0.45	0.40
	0.3	0.032	0.019	0.014	0.010	0.55	0.43	0.38	0.33
	0.4	0.024	0.014	0.010	0.007	0.50	0.39	0.35	0.30
	0.5	0.020	0.012	0.009	0.006	0.45	0.37	0.32	0.27
	1.0	0.012	0.006	0.005	0.003	0.37	0.30	0.24	0.21
	2.0	0.006	0.003	0.002	0.002	0.28	0.22	0.19	0.16
	4.0	0.004	0.002	0.002	0.001	0.27	0.20	0.17	0.14

(c) *BIS Code Method*

Bureau of Indian Standard Code IS 3370 (Part IV) -1967 gives tables for BM and hoop tension in circular tanks for various condition of joints and various types of loading. However, in this section only the case of circular tank with rigid base subjected to triangular water pressure will be discussed. Table 22.7 gives the coefficients for hoop tension at various height in the wall for various values of $\frac{H^2}{D t}$ ratio. The hoop tension is calculated from the expression :

$$\text{Hoop tension, } T = (\text{coeff.})\ \gamma H \ \frac{D}{2}$$

(d) *Approximate Method*

In Figure 22.7, triangle ABC shows hydrostatic water pressure on wall AB. In the approximate method, it is assumed that cantilever action will be in a height h above base i.e., height BD. The assumed distribution of load causing

cantilever bending and hoop action is shown in Figure 22.7, thus showing that the hoop tension will be maximum at the height h above base i.e., at point D. The value of height h is calculated approximately as follows :

$$h = \frac{H}{3} \text{ or 1 m (whichever is more)for } 6 \leq \frac{H^2}{D\,t} \leq 12$$

$$= \frac{H}{4} \text{ or 1 m (whichever is more)for } 12 < \frac{H^2}{D\,t} \leq 30$$

The cantilever BM and hoop tension can now be easily calculated at any height from the load distribution diagram. Their maximum values will be :

Maximum cantilever BM, $M = \gamma\ H\ \frac{h^2}{6}$

Maximum hoop tension at D, $T = \gamma\ (H - h)\ \frac{D}{2}$

The reinforcement for cantilever BM will have to be provided on water face, whereas, hoop reinforcement will be in the form of rings to be provided either in the middle of the thickness of wall or on both faces.

iii. Rectangular Tank

For small capacity sometimes a rectangular tank is adopted to avoid exessive expenditure on curved shuttering required for circular water tanks. These tanks, however, are uneconomical for large capacity. The walls of a rectangular water tank are subjected to vertical as well as horizontal BM and pull on some portion of walls. The top edge of the walls which supports a relatively light roof slab can be treated as hinged or free, if the tank is open. The bottom edge of the walls which is normally built integrally with the base slab is treated as fixed. There are situations where master pads, etc. are provided between the wall and the bottom slab, then the joint is treated as hinged.

The analysis of moments in walls of the tank is more difficult as the water pressure applies a triangular load on them. The magnitude of moments will depend upon the relative proportions of length, width and height of the tank and the support conditions of the top and bottom edges of the walls. The analysis of moments in the walls of a tank is made by elastic theory. The resulting differential equation is not easy to solve and therefore accurate solutions covering all cases are not available. IS 3370 (Part IV) gives tables from which moments and shears in walls for certain edge conditions can be calculated either directly or with suitable modifications. Alternatively, an approximate method can be employed for the design of open rectangular tanks. The method is discussed below.

Approximate Method

Consider an open rectangular tank of dimension L (Length) × B (Breadth) × H (Height). For designing by approximate method, the tank can be divided into two categories :

(a) $L/B \leq 2$

(b) $L/B > 2$

(a) L / B <= 2 : In Figure 22.9(a), triangle MPQ shows hydrostatic water pressure on wall MP. It is assumed that the cantilever action will be in a height h above base. The assumed distribution of load causing vertical cantilever bending and horizontal bending is shown in Figure 22.9(a), which shows that the horizoiltal bending will be maximum at the height h above base. The value of height h is taken as H //44 or 1 m whichever is greater. The final horizontal BM is calculated by the analysis of the continuous frame shown in Figure 22.9(b). The moment distribution method can be employed for the purpose. The cantilever BM at the base can be easily calculated as below :

$$\text{Vertical cantilever BM at the base} = \gamma\ H\ \frac{h^2}{6}$$

In addition to the BM, the walls are also subjected to direct tension whose values at height h above base are as follows :

$$\text{Direct tension in long walls} = p\ \frac{B}{2}$$

$$\text{Direct tension in short walls} = p\ \frac{L}{2}$$

where, p = hydrostatic pressure at height h above base $= \gamma\ (H - h)$

(b) L / B > 2 : The long walls are assumed to bend vertically as cantilever under the action of triangular hydrostatic pressure. In short walls, cantilever action is assumed in a height h above base. The distribution of load causing cantilever action and horizontal bending is same as for L / B I 2. The bending moments can thus be calculated easily as given below :

$$\text{Maximum cantilever BM in long wall} = \frac{\gamma\ H^3}{6}$$

$$\text{Maximum cantilever BM in short wall} = \frac{\gamma\ H\ h^2}{6}$$

Maximum horizontal BM at a height h above base in short wall

$$\approx \frac{p\ B^2}{2} \quad \text{at ends as well as mid}$$

In addition to the bending moments, the long and short walls are subjected to direct tension. Since the short walls are assumed to be supported on long walls at its ends, there will be pull in the long walls which will be maximum at height h above base and can be calculated as below :

$$\text{Maximum tension in long wall} = \frac{p\ B}{2} \text{ where, } p = \gamma\ (H - h)$$

As the long walls behave as cantilever, therefore, no pull is transmitted to short wall because of water pressure on long walls. However, the water pressure on the end 1 m width of long wall is assumed to cause direct tension in short wall because cantilever action will not be there close to the ends. Thus,

$$\text{Maximum direct tension in short wall at height } h \text{ above base} = \frac{p\ L'}{2}$$

where, $L' = 2$ m

Design of section subjected to the combined effects of bending and pull is described in the following :

Let M' be the horizontal BM and T be the pull in it. The pull in the wall is transmitted through steel bars thus introducing BM in wall which will thereby reduce the BM. The reduced BM will be

$$M = M' - Tx,$$

where, x = distance of steel from central axis = $d - t/2$,

d = effective depth of section,

t = thinckess of wall.

Area of steel is calculated separately for net BM and for the pull by the following expressions :

$$\text{Area of steel for net BM, } A_{s1} = \frac{M' - Tx}{\sigma_{st}\ j\ d}$$

$$\text{Area of steel for net pull, } A_{s2} = \frac{T}{\sigma_{st}}$$

$$\text{Total steel, } A_s = A_{s1} + A_{s2}$$

NUMERICALS

Desgin an open circular water tank resting on firm ground with flexible base for 350 kl capacity.

Solution

(1) *Proportioning of the Tank*

Assuming the height of wall (inside) as 4.0 m.

Keeping 200 mm free board, effective depth of water, $H = 4.0 - 0.2 = 3.8$ m

Let D be the inside diameter of the tank in meter.

or, $\frac{\pi}{4} D^2 \times 3.8 = 350$

or, $D = 10.83$ m

Providing 11.0 m diameter ($r = 5.5$ m).

(2) *Allowable Stresses*

M 20 grade concrete and Fe 415 grade steel (HYSD) will be used.

Permissible direct tensile stress of concrete, $\sigma_{ct} = 1.2$ MPa

Permissible tensile stress of steel,

$\sigma_{st} = 150$ MPa upto 225 mm from water face

$= 190$ MPa beyond 225 mm from water face

(3) *Design of Wall*

Height of wall, $h = 4.0$ m

Hoop tension at the base of the wall, $T_1 = \gamma r h = 10 \times 5.5 \times 4.0 = 220$ kN

Hoop tension at 3.0 m from top, $T_2 = 10 \times 5.5 \times 3.0 = 165$ kN

Total hoop tension in the bottom 1 m depth, $T = \frac{1}{2}(T_1 + T_2) = 192.5$ kN

Area of steel required in the bottom 1 m depth

$$A_s = \frac{T}{\sigma_{st}} = \frac{192.5 \times 10^3}{150} = 1284 \text{ mm}^2$$

Provide 7 ϕ 16, A_s (provided) $= 7 \times 201 = 1407$ mm^2

Tensile stress in concrete $= \frac{T}{A_c + (m-1) A_s} \leq \sigma_{ct}$

or, $\frac{192.5 \times 10^3}{A_c + (13-1) \times 1407} \leq 1.2$ or, $A_c = 143533 \text{ mm}^2$

Thickness required, $t = \frac{A_c}{1000} = 143.5$ mm

Provide the thickness of wall as 160 mm at the base and tapered to 100 mm at the top.

Average thickness between 3.0 and 4.0 m depth = $100 + \frac{160 - 100}{4} \times 3.5$

$= 152.5 \text{ mm} > 143.5 \text{ mm}$

The hoop reinforcement in vertical wall is given in Table 22.3.

Table 22.3 : Hoop Reinforcement in Vertical Wall

Distance from Top (m)	Hoop Tension T (kN)	Reinforcement	
		Required (mm^2)	Number of ϕ 12 Bars
0.0 to 1.0	27.5	183	4
1.0 to 2.0	82.5	550	5
2.0 to 3.0	137.5	917	9
3.0 to 4.0	192.5	1284	12

Average thickness of wall = 130 mm

Minimum percentage of steel for 130 mm thickness

$= 0.24 - \frac{(130 - 100)}{350} \times 0.08$

$= 0.233$

Minimum steel reinforcement = 0.233/100 * 1000 * 130

Provide ϕ10 @ 250 mm c/c vertical steel.

As the thickness of wall is less than 225 mm, therefore, reinforcement has been provided in the middle of the thickness of wall.

(4) *Design of Base Slab*

As the slab is resting on the ground, provide a nominal thickness of 150mm.

Minimum steel, $A_s = \frac{0.24}{100} \times 1000 \times 150 = 360 \text{ mm}^2/\text{ m}$ in each direction

Provide half steel on each face.

Provide ϕ 8 @ 250 mm c/c in both directions and at top and bottom.

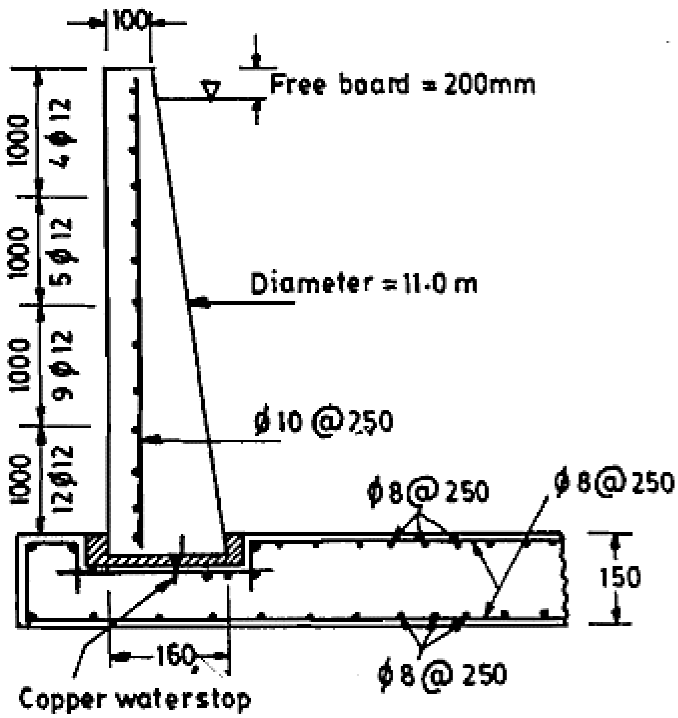


Figure 22.5 : Reinforcement in Tank Wall and Base Slab
(All dimensions are in mm)

Q. Redesign the tank of above example, assuming that the base of the wall is monolithic with the base slab.

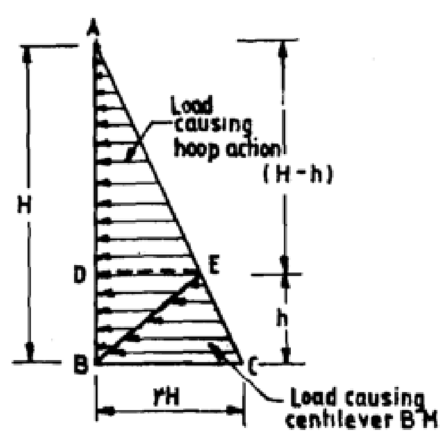
Solution

(1) *Dimensions of the Tank*

Diameter, D =11.0 m (r =5.5 m)

Height, H = 4.0 m

Providing thickness of wall, t =150 mm



Assumed Load Distribution in Circular Tank with Rigid Base

(2) *Allowable Stresses and Design Constants*

Grade of concrete =M 20

Grade of steel = Fe 415 (HYSD)

Permissible direct tensile stress of concrete, $\sigma_{ct} = 1.2$ MPa

Permissible bending compressive stress of concrete, $\sigma_{cbc} = 7.0$ MPa

Modular ratio, $m = \dfrac{280}{3\,\sigma_{cbc}} = 13$

Permissible tensile stress of steel,

$$\sigma_{st} = 150 \text{ MPa upto 225 mm from water face}$$
$$= 190 \text{ MPa beyond 225 mm from water face}$$

N. A. depth coefficient, $n = \dfrac{1}{1+\dfrac{\sigma_{st}}{m\,\sigma_{cbc}}} = \dfrac{1}{1+\dfrac{150}{13\times 7}} = 0.378$

Lever arm coefficient, $j = 1 - n/3 = 0.874$

$$k = \frac{1}{2}\sigma_{cbc}\, n\, j = 1.155$$

(3) *Design of Wall for Cantilever Action*

Employing approximate method of analysis.

$$\frac{H^2}{D\,t} = \frac{4.0^2}{11.0 \times 0.15} = 9.7$$

Cantilever action will be upto h, where, $h = H/3$ or 1 m whichever is greater.

$$\therefore\ h = \frac{4}{3} = 1.333 \text{ m}$$

Maximum cantilever BM,

$$M = \frac{1}{6}\gamma H h^2 = \frac{1}{6} \times 10 \times 4.0 \times 1.333^2 = 11.85 \text{ kN m/m}$$

Effective thickness of wall required = $\sqrt{\dfrac{M}{Kb}} = \sqrt{\dfrac{11.85 \times 10^6}{1.155 \times 1000}} \approx 102$ mm

Effective thickness allowable, $d = 150 - 35 = 115$ mm > 102 mm (O. K.)

Area of vertical steel required,

$$A_s = \frac{m}{\sigma_{st}\, j\, d} = \frac{11.85 \times 10^6}{150 \times 0.874 \times 115} = 786 \text{ mm}^2\text{/m}$$

Provide $\phi 12$ @ 140 mm c/c ($A_s = 807 \text{ mm}^2$) upto 1.4 m from base and above this height curtail half of the bars. So, steel in the top 2.6 m depth will be ϕ 12 @ 280 mm c/c ($A_s = 404 \text{ mm}^2$). These vertical bars are to be provided at inner face.

Minimum percentage of steel for 150 mm thickness

$$= 0.24 - \frac{150 - 100}{350} \times 0.08 = 0.23$$

$$\text{Minimum steel} = \frac{0.23}{100} \times 1000 \times 150 = 345 \text{ mm}^2 < 404 \text{ mm}^2 \quad \text{(O. K.)}$$

(4) *Design of Wall for Hoop Tension*

Maximum hoop tension,

$$T_1 = \gamma\, r\, (H - h) = 10 \times 5.5 \times (4.0 - 1.333) = 146.7 \text{ kN at } 1.333 \text{ m from base}$$

Hoop Tension at 2.0 m for top, $T_2 = 10 \times 5.5 \times 2.0 = 110.0$ kN

Total hoop tension between 2.0 m from top to 2.667 m from top,

$$T = \frac{1}{2}(T_1 + T_2)$$

$$= 128.4 \text{ kN}$$

$$\text{Area of hoop steel, } A_s = \frac{T}{\sigma_{st}} = \frac{128.4 \times 10^3}{150} = 856 \text{ mm}^2$$

(provide 12 ϕ 10, $A_s = 942 \text{ mm}^2$)

The hoop steel in vertical wall is given in Table 22.10.

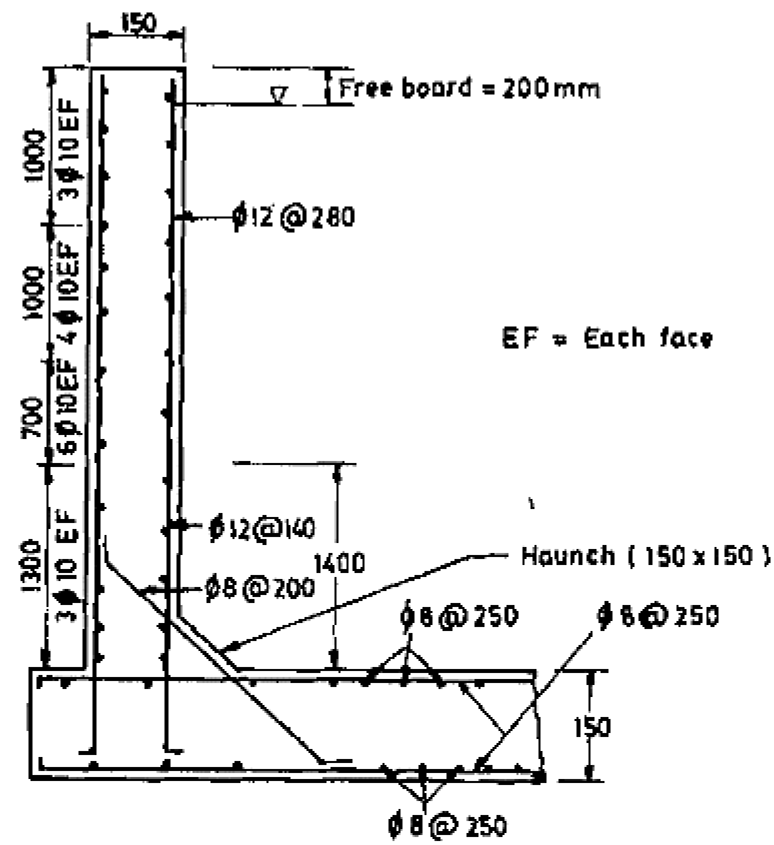
Table 22.10 : Hoop Steel in Vertical Wall

Distance from top (m)	Hoop Tension T (kN)	Reinforcement	
		Required (mm2)	Number of ϕ 10 bars
0.0 to 1.0	27.5	183	3 on each face (min)
1.0 to 2.0	82.5	550	4 on each face (min)
2.0 to 2.7	128.4	856	6 on each face (min)
2.7 to 4.0	--	404 (min)	3 on each face (min)

Tensile stress in concrete

$$= \frac{T}{A_c + (m-1)A_s} = \frac{128.4 \times 10^3}{667 \times 150 + (13-1) \times 942} = 1.15 \text{ MPa} < 1.2 \text{ MPa (O.K.)}$$

Provide 150 × 150 mm haunches at the junction of wall with the base slab and provide ϕ 8 @ 220 mm c/c in it as shown in Figure 22.8.



(5) Design of Base Slab: Same as previous example.

Q.2 RECTANGULAR WATER TANK DESIGN

Design a rectangular RC water tank (resting on the ground) with an open top for a capacity of 80 000 litres. The inside dimension of the tank may be taken as 6 m × 4 m. Design the side walls of the tank using M 20 grade concrete and Fe 250 grade I mild steel. Draw the following views:

- **Cross-sectional elevation of the tank showing reinforcement details in tank walls.**
- **Plan of the tank showing reinforcement details.**

1. Data

Capacity of tank	=	80 000 litres
Size of tank	=	6 m × 4 mm
Free board	=	150 mm
Materials:		M 20 grade concrete
		Fe 250 grade I mild steel

$\sigma_{cb} = 7\ \text{N/mm}^2$

$\sigma_{st} = 115\ \text{N/mm}^2$ (on faces near water face)

$\sigma_{st} = 125\ \text{N/mm}^2$ (on faces away from water face)

$m = 13,\ Q = 1.41,\ j = 0.84$

2. Dimension of tank

Referring to Fig. 5.4,

Height of water $= \left(\dfrac{80\,000 \times 10^3}{600 \times 400}\right) = 3.35$ m

Free board = 150 mm

Height of side walls $H = (3.35 + 0.15) = 3.5$ m

$(L/B) = (6/4) = 1.5 < 2$

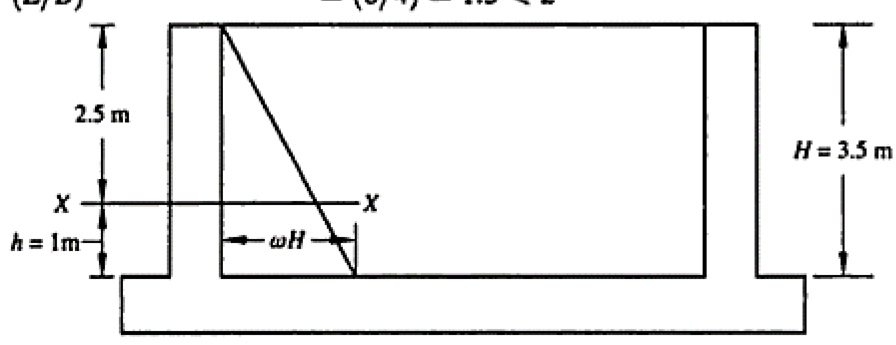


Fig. 5.4 Rectangular water tank

It is assumed that the walls function as a continuous slab subjected to water pressure above $(H/4)$ or 1 m from the bottom, and as a cantilever for the bottom 1 m.

Therefore intensity of pressure $p = \omega(H - h)$ at XX

$= (10 \times 2.5) = 25\ \text{kN/m}^2$

Alternatively, the design Tables of IS:3370 (Part-IV)-1967, clause 2.2 can be used for computation of moments in tank walls.

3. Moments in side walls

The moments in side walls are determined by moment distribution

$L = 6$ m, $B = 4$ m

$$\left(\frac{pL^2}{12}\right) = \left(\frac{25 \times 6^2}{12}\right) = 75 \text{ kNm}$$

$$\left(\frac{pL^2}{8}\right) = \left(\frac{25 \times 6^2}{8}\right) = 112.5 \text{ kNm}$$

$$\left(\frac{pB^2}{12}\right) = \left(\frac{25 \times 4^2}{12}\right) = 34 \text{ kNm}$$

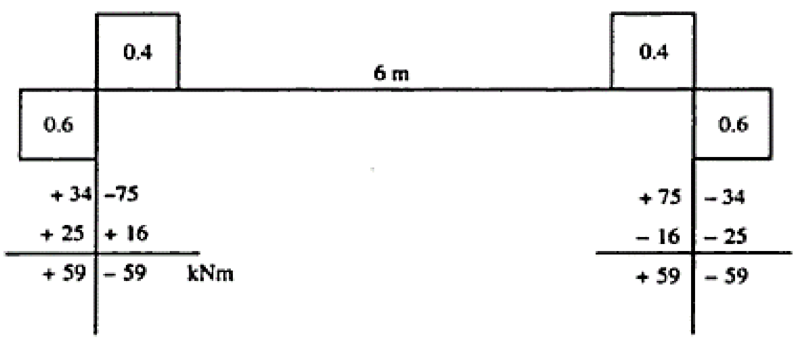
$$\left(\frac{pB^2}{8}\right) = \left(\frac{25 \times 4^2}{8}\right) = 50 \text{ kNm}$$

The moment distribution together with the BM diagram is shown in Fig. 5.5

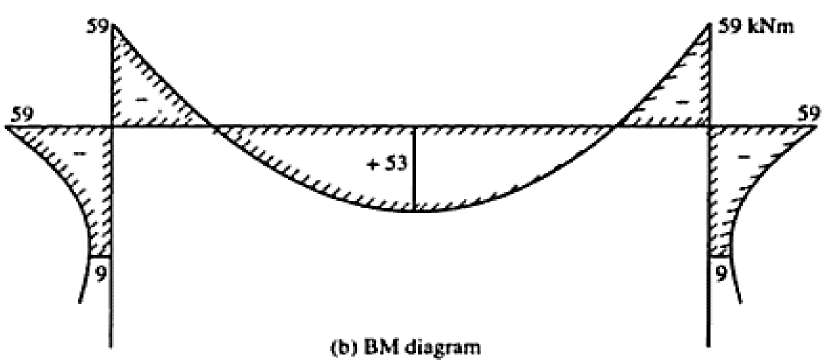
Direct tension in short wall $T \quad = (0.5 \times 25 \times 6) = 75$ kN

Referring to Fig. 5.6,

$$A_{st} \text{ (long wall corners)} = \left(\frac{M - Tx}{\sigma_{st} j d}\right) + \left(\frac{T}{\sigma_{st}}\right)$$



(a) Moment distribution



(b) BM diagram

Fig. 5.5 Moments in tank walls

Therefore $A_{st} = \left[\frac{(59 \times 10^6) - (50 \times 10^3 \times 90)}{100 \times 0.84 \times 215}\right] + \left[\frac{50 \times 10^3}{100}\right]$

$= 3480 \text{ mm}^2$

Spacing of 20 mm diameter bars $= \left(\frac{1000 \times 314}{3480}\right) = 90$ mm c/c

Adopt 20 mm diameter bars at 80 mm c/c ($A_{st} = 3928 \text{ mm}^2$)

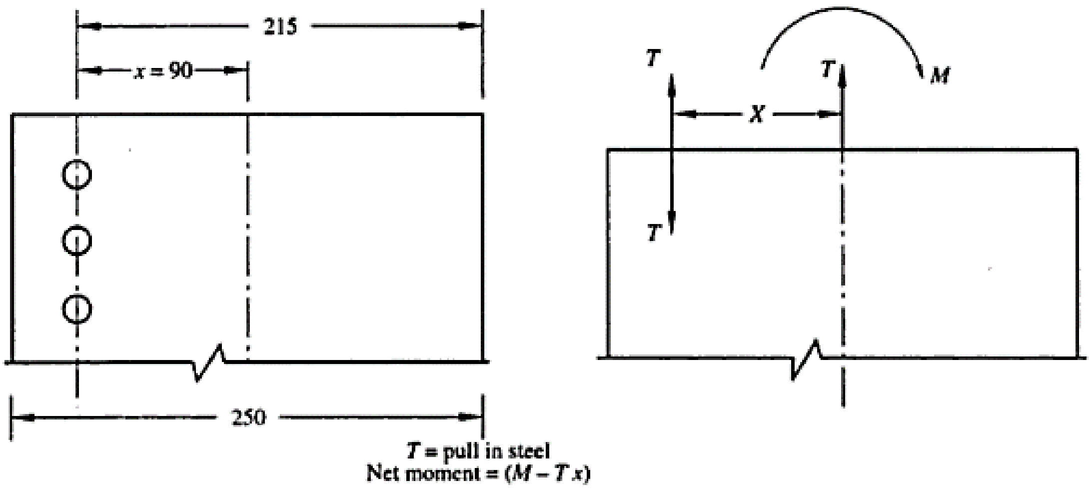


Fig. 5.6 Moment and direct tension in tank walls

Reinforcement at centre of span (long walls)

$$= \left[\frac{(53 \times 10^6) - (50 \times 10^3 \times 90)}{125 \times 0.84 \times 215}\right] + \left[\frac{50 \times 10^3}{125}\right] = 2500 \text{ mm}^2$$

Half the bars from the inner face at support are bent towards the outer face at the centre, providing an area of $(0.5 \times 3928) = 1964 \text{ mm}^2$. For the remaining area of $(2500 - 1964) = 536 \text{ mm}^2$, provide 16 mm diameter bars at 150 mm c/c. For short walls, bend 50% of the bars towards the outer face at the centre.

5. Reinforcement for cantilever moment

(For 1 m height from the bottom)

Cantilever moment $= (3.5 \times 10 \times 1/2 \times 1/3) = 5.833$ kNm

Therefore $A_{st} = \left(\frac{5.833 \times 10^6}{100 \times 0.84 \times 215}\right) = 323 \text{ mm}^2$

Minimum reinforcement $= 0.3\% = \left(\frac{0.3 \times 1000 \times 250}{100}\right)$

$= 750 \text{ mm}^2$

Reinforcement on each face $= (0.5 \times 750) = 375 \text{ mm}^2$

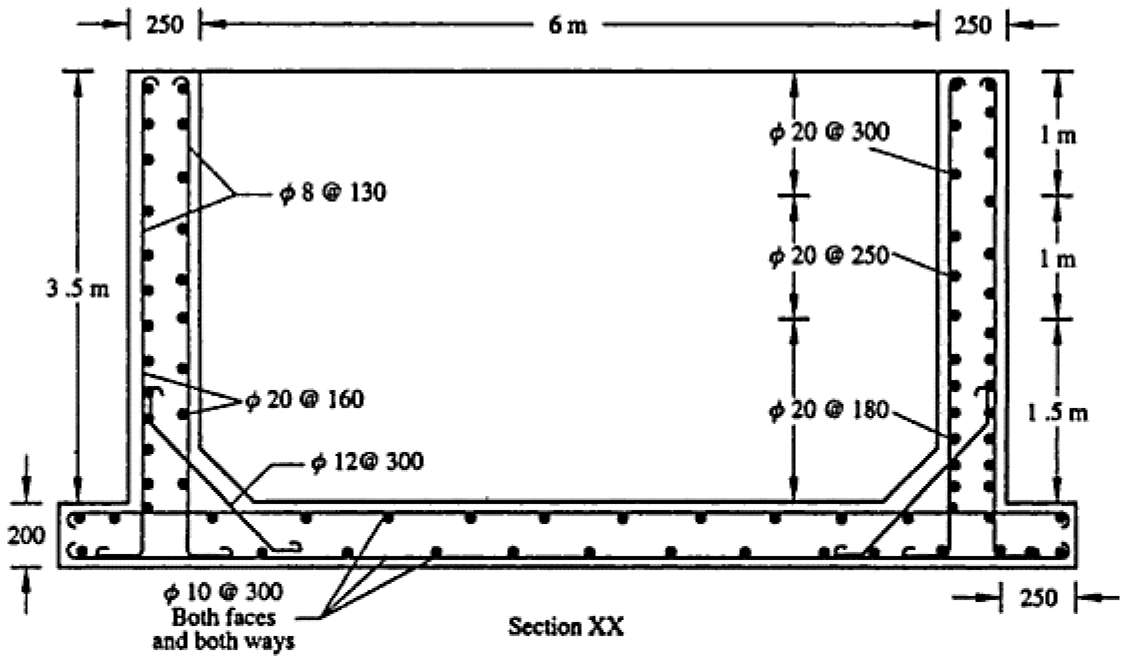
Spacing of 8 mm diameter bars $= \left(\frac{1000 \times 50}{375}\right) = 130 \text{ mm c/c}$

Adopt 8 mm diameter bars at 130 mm c/c on both faces.

6. Base slab

The base slab rests on the ground. Provide 200 mm base slab with 10 mm diameter bars at 300 mm c/c, both ways on each face.

The reinforcement details in the rectangular tank are shown in Fig. 5.7.



Section XX

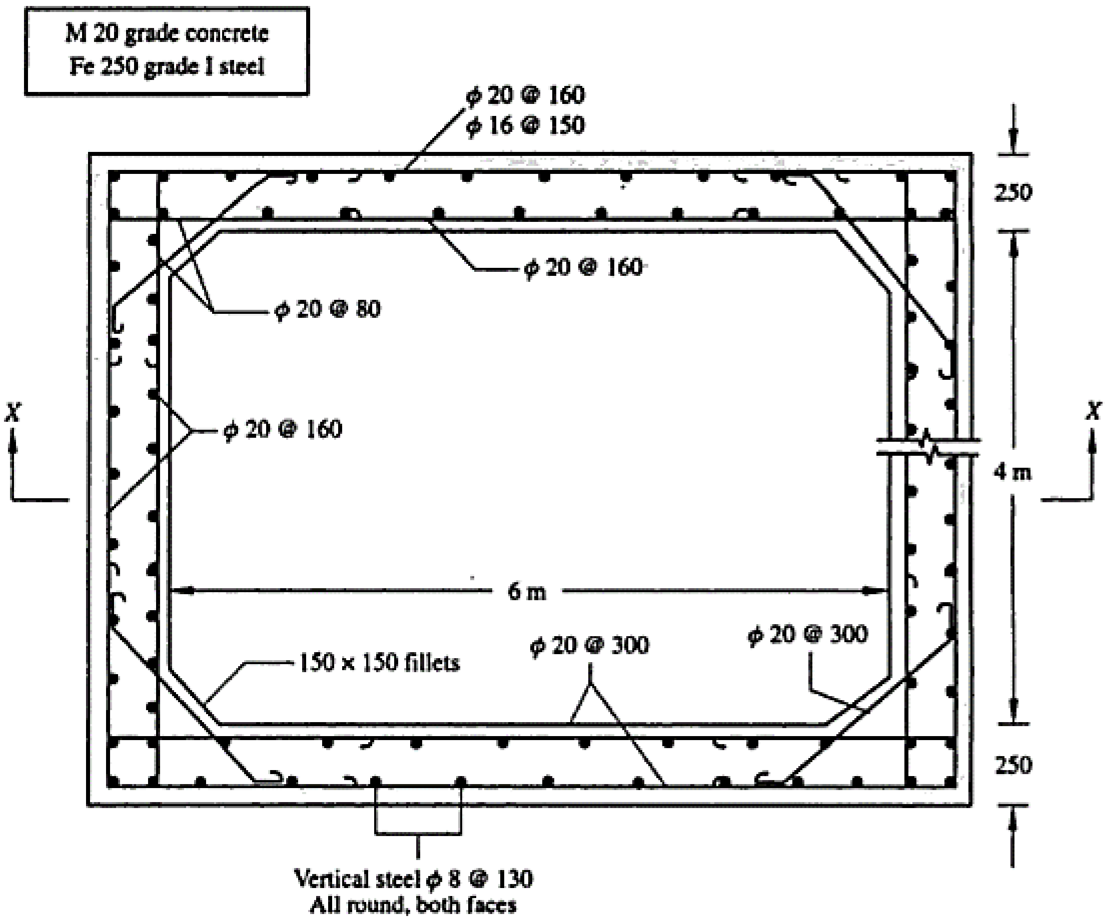


Fig. 5.7 Reinforcement details in rectangular tank

Numerical 3:

Design the sidewalls of a rectangular reinforced concrete water tank of dimensions 6 m by 2 m and having a maximum depth of 2.5 m, using M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Size of tank = 6 m by 2 m
Length = L = 6 m and breadth = B = 2 m
Depth of tank = H = 2.5 m
Materials: M-20 grade concrete and Fe-415 HYSD bars

b) Permissible Stresses

$\sigma_{cb} = 7\ N/mm^2$ $\quad Q = 1.20$
$\sigma_{st} = 150\ N/mm^2$ $\quad j = 0.86$

c) Design of Long walls

$L = 6$ m and $B = 2$ m
(Ratio $(L/B) = (6/2) = 3 > 2$

Long walls are designed as vertical cantilevers and short wall as a slab spanning horizontally between long walls.

Maximum bending moment at base of long wall is computed as,

$$M_L = \left(\frac{wH^3}{6}\right) = \left(\frac{10 \times 2.5^3}{6}\right) = 26.04 \text{ kN.m}$$

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{26.04 \times 10^6}{1.20 \times 10^3}} = 147.3 \text{ mm}$$

Adopt effective depth $= d = 150$ mm and overall depth $= D = 180$ mm

$$\therefore \quad A_{st} = \left(\frac{26.04 \times 10^6}{1.20 \times 10^3}\right) = 1340 \text{ mm}^2$$

Provide 16 mm diameter bars at 150 mm centres ($A_{st} = 1341$ mm^2) at the bottom of the tank. Spacing increased to 170 mm for the top 1 m portion of the tank.

Intensity of pressure 1 m above the base is computed as

$$P = w(H - h) = 10(2.5 - 1) = 15 \text{ kN/m}^2$$

Direct tension in long walls $= T_L = [(15 \times 2)/2] = 15$ kN

$$\therefore \quad A_{st} = \left(\frac{15 \times 10^3}{150}\right) = 100 \text{ mm}^2$$

Minimum area of steel $= 0.3\% = (0.003 \times 180 \times 1000) = 540$ mm^2

$$\text{Spacing of bars} = \left(\frac{1000 \times 79}{540}\right) = 146 \text{ mm}$$

Since steel is distributed on both faces, provide 10 mm diameter bars at 280 mm centres on both faces in the horizontal direction.

d) Design of Short walls

Intensity of pressure = p = 15 kN/m^2
Effective span of horizontally spanning slab = (2 + 0.18) = 2.18 m

$$\text{Bending moment (corner section)} = \left(\frac{pL^2}{12}\right) = \left(\frac{15 \times 2.18^2}{12}\right) = 5.94 \text{ kN.m}$$

Tension transferred per metre height of short wall = (15 × 1) = 15 kN

$$\therefore \quad A_{st} = \left[\frac{M - T.x}{\sigma_{st}\, j\, d}\right] + \left[\frac{T}{\sigma_{st}}\right]$$

$$A_{st} = \left[\frac{(5.94 \times 10^6) - (15 \times 10^3)(150 - 90)}{(150 \times 0.86 \times 150)}\right] + \left[\frac{15 \times 10^3}{150}\right] = 360 \text{ mm}^2$$

Minimum reinforcement = 0.3 percent = (0.003 × 180 × 1000) = 540 mm^2
Hence, provide 10 mm diameter bars at 280 mm centres on both faces in the horizontal direction with an effective cover of 30 mm

e) Design for Cantilever effect of short wall

Maximum bending moment at bottom of wall is computed as,

$$M = (0.5 \times 10 \times 2.5 \times 1 \times 0.333) = 4.2 \text{ kN.m}$$

Effective depth using 10 mm diameter bars = (180 − 40) = 140 mm

$$A_{st} = \left(\frac{4.2 \times 10^6}{150 \times 0.86 \times 140}\right) = 232 \text{ mm}^2$$

Minimum reinforcement = 0.3% = (0.003 × 180 × 1000) = 540 mm^2
Provide 10 mm diameter bars at 280 mm centres in the vertical direct on both faces.
The details of reinforcements in the tank walls are shown in Fig. 15.12.

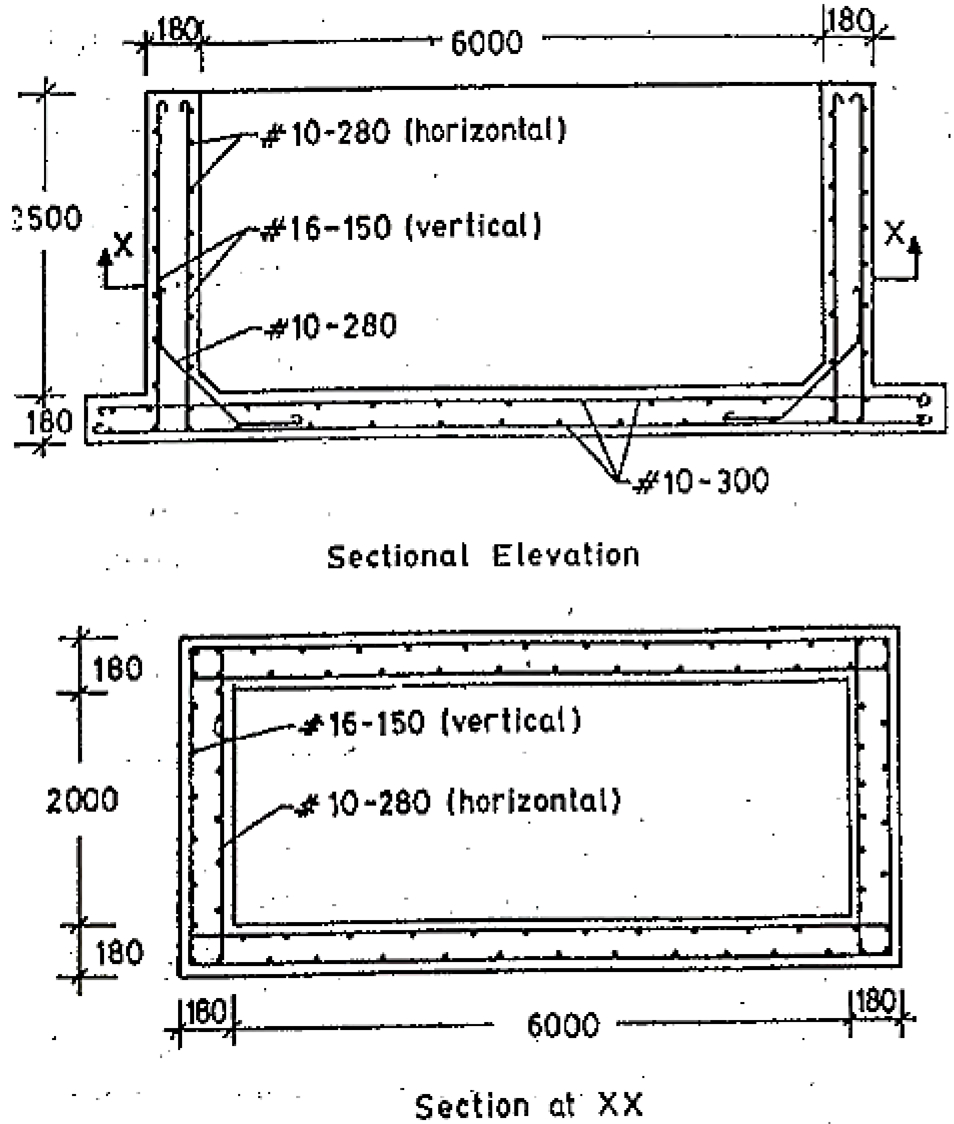


Fig. 15.12 Reinforcement Details in Rectangular Tank